MOORES ARITHMETICK:

Discovering the *secrets* of that *Art*, in *Numbers* and *Species*.

In two Bookes.

The first teaching (by Precept and Ex ample) the ordinary Operations in Numbers whole and broken; the *Rules* of *Practise*, Int rest, and performed in a more facile manner by *D cimalls*, then hitherto hath been published; th excellency, and new practise and use of the *Log rithmes*, *Nepayres bones*, together with many ne *Propositions*, touching the *Quantities*, *Qualit* *Resultments*, and *Rules* of *Medicines*.

The second, the great *Rule* of *Algebra* in *Species*, r solving all *Arithmeticall* Questions by *supposition*.

With a Canon of the Powers of Numbers.

Fitted to the meanest Capacity, and published for th generall good of this Kingdome.

By JONAS MOORE, late of *Durham*.

London, Printed by *Thomas Harper* for *Nathani Brookes*, at the Angell in Cornehill, 1650.

Thomas Spurhne

To the Honourable,

Sir VVILLIAM PERSALL, Kt.
EDMUND VVILD, Esq. and
NICHOLAS SHUTTLEVVORTH, Esq.

The Authour,

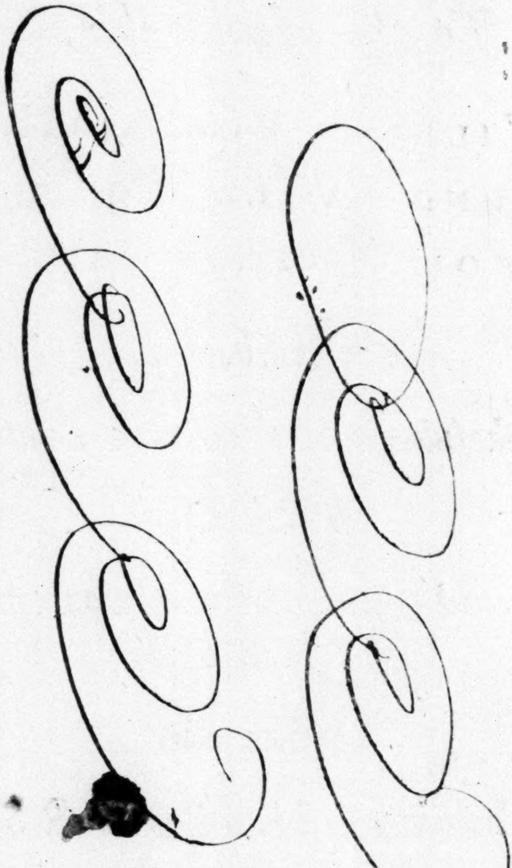
In *thankefulnesse* of their great *Curtesies,*

AND

Forward *Bounty* to the *Advancement* of *ART*:

Dedicateth

These his *first* LABOURS.



The Epistle to the Reader.

Reader:

TO trouble thee with the due praises and Benefit of the Noble Art *of* Arithmetick, *were but* actum agere, *or lost labour, whilst so many have done it before, and the manyfold uses of it to* Mankinde *are knowne almost to every one*; *neither shall I excuse the publishing heereof by the commandment of* Superiours, *intreaty of* Friends, *&c.* No, *it is for the publique good, and learning the speedy practice of this Art, I make this Piece now come to thy hands.*

I have to my power observed this Rule; To write nothing that might be superfluous, nothing that should be difficult or obscure; *and*

concerning the censure *of any, if it be of the* Judicious, *I dare abide it, if of some* criticall Foole, *having no* Genius *to these Arts, and for ostentation sake, will make a shew of* knowledge *amongst the ignorant, detracting from other mens labours, (and a number of such there be) I value him not, this booke is above his* Calumny ; *but for thy cause* loving Friend , *that desires to profit by it, I writ it upon the reasons following.*

Vpon the first comming in of the Scots 1640. *in a* solitary retyrednesse, *with a settled resolution, I fell upon the studyes* Mathematicall , *animated thereunto by the promised helpe of Mr.* William Milburne *Minister of* Brancepeth *in the County of Durham , my most worthy friend, and a great Master in all parts of* Learning , *who not many weekes after departed this life, leaving me either in choise to give over my journey, or travell without either Guide or* Company ; *and a long time did I wander in the* by-paths *of other mens* Mechanicall *Practises, till at last by a most happy accident I had Mr.* Oughtreds Clavis Mathematicæ *bestowed upon me , by which I unlocked the Mysteryes of the Demonstrations of the Auncients , and set my selfe in the high way to*

Perfection,

Perfection; *unto which Booke, and to the Authors most absolute favours, I owe all the* Mathematicall *knowledge I have. Now these rubs and difficulties I met with in the way, I have made even and cleare, and as* I *thought my selfe bound, have delivered thee a most compendious and easie Method towards the attaining the favour of the first* Mistres *that must be saluted,* Arithmetick, *and have fitted it to the use of every one with these two Cautions.*

1. *If thou intend so much as may fit thee onely for Trade, &c. then mayest thou onely peruse the ordinary Rules whereof there are plenty for thee, and omit the more difficult places.*

2. *But if thou intend to make this peece the* way *to thy further* Progresse *in these studies, then use a little more paines, omit nothing, and doubt not of an answerable performance and satisfaction to thy expectation; and let it be no prejudice to this booke that the* Author *at present professeth to teach the Mathematicks, and therefore it should be made by him more difficult or obstruse for his owne ends; no, for herein I am of a contrary opinion to such ordinary* Mechanicks, *who judge by their owne humours; for* I *value not how*

much

much any one knowes before I *meddle, or how much any profit in never so little time: being confident the abler any one growes, his pleasure is more vehement, and desire more unsatisfied, till he attaine to his wished for perfection.*

I *had intended to have added hereto the* demonstrations *of the severall rules after made use of: as also to the second part a Method for reducing Æquations of an higher to a lower degree, and of adfected to simple powers; but it is now two years since this peece was delivered to be printed, and these peradventure would have hindred thee from the benefit of the other; therefore as it is, take it in good part, and if the times serve, the charge be not too great, and* I *finde thy kinde acceptation hereof, expect these following Treatises to be published, the most whereof are perfected for the Presse.*

1. The perfect Geometer.

Containing the six first Bookes of Euclid, *so much of the* 11, 12, *and* 13. *as concerne the knowledge of solids; symbolically demonstrated in a very compendious forme.*

2. Locus

2. Locus resolut.

Containing Euclids Data, *with sundry propo-ions analitically invented, and Geometrically solved and demonstrated.*

3. The Mechanick.

Containing the practice of Geometry in survey-ng, fortification, Architecture, &c.

Via ad Tubi optici, speculi, ustorii, necnon Instrumenti auditorii perfectionem aperta.

Containing the Doctrine *of* Comicall Secti-ons, *and demonstrating the nature of such* bodies *as must serve to the former purpose.*

5. Astronomia Britanica.

Containing the uses of the Globes *and their* Projections, *the* Theory *of the* Planets, *Ancient and Moderne; together with* Astronomicall Ta-bles, *for the calculating the places of the Planets,*

and

and for Ecclipses, made after a new manner, a[...] far more easie for use, then any extant.

Lastly, without the staine of ingratitude, I ca[...] not but mention to the World the great favours [...] have received from the truely noble paire of Br[...] thers, Richard Shuttleworth *of* Galthrop *in th[...] County of* Lancaster, *Esquire*, *and* Nichol[...] Shuttleworth *of* Facett *in the County of* York[...] *Esquire, not only in the furtherance of me in the[...] studies; but also in other my urgent affaires, an[...] but for whom this piece had been* abortive, *it bein[...] gotten, borne, and nourished by their especiall in[...] fluence and protection.*

Reader, so desiring a prosperous successe to the[...] in all thy studies: I shall ever remaine a Servan[...] to all true Lovers of Art,

From my Chamber at Mr. Elias Allen, *his house over against St.* Clements *Church in the Strand.* 30th. *of* October, 1649.

JONAS MOORE.

A Catalogue of the Chapters contained in the first Part.

Chap. 15.

A Catalogue of the Chapters contai[-]ned in the second Part.

Reader, for the literall, and some other mistakes, it will be in thy power to correct.

As Page 88. line 14. reade *to twice the fifth,* also p. 89. l. 13. reade for *first, fifth, fifth, first,* and the line following *doubled into the third,* the rest followeth at the end. p. 172. *Camphuratum* for *Campla.*

Chap. I.

Chap. I.

A short *Introduction* into the parts of *Arithmetique*.

1. ARithmetique is the art of numbring well.

The *Subject* of *Arithmetique* is *Number*, as that unto which all the precepts and doctrine of the Art of computation (whether by numbers whole, or broken) hath a particular relation.

2. *Arithmetique* is either *simple* or *comparative*.

3. *Simple* is that which considereth the simple nature of *Numbers*.

4. *Number* is that, according to which any thing is numbered, whether it be corporeall or incorporeall.

According to an unit any thing is said to be one, as one God, one wife, one dish of meat, &c. [This is according to *Ramus*: the Ancients say, that

unity is the beginning of *Number*, yet no *Number.*

According to the number, two things are ſaid to be two, two ſtarres, two men, &c.

5. *Number* is either *whole* or *broken*.

Whole we call *Integers*, from the Latin word *Integrum*, and broken Numbers *Fractions*.

6. A whole *Number* is either of *unity*, or of *multitude*.

7. An unity or *unite* is the beginning of *multitude*.

8. *Multitude* is the collection of *unites*, as 1,1,1,1,1,1,1,1,1,1. are equall to 10. being collected together.

9. *Fractions* ariſe from the diviſion of the unite into parts. Number ſimply conſidered cannot be leſſe then unity. But when it ſignifieth any other thing or *Quantity*, then in reſpect of that Quantity, it may be divided into leſſer parts, then that one Quantity: As one yard being divided into four equall parts, the half of that yard is leſſe then one, and is ſet downe as a fraction thus, $\frac{1}{2}$.

10. In Numbers of any ſort, two things are to be conſidered, $\left\{\begin{array}{l}\textit{Notation,}\\ \textit{Numeration.}\end{array}\right.$

11. *Notation* teacheth how to deſcribe any number by certaine notes or characters, and to declare the value thereof being ſo deſcribed.

This *Notation* is that which ſome called *Numeration*.

12. But *Numeration* (here) is that, which teacheth

cheth how from Numbers given to finde out another required, and which (as is afterwards declared) consists in the *Composition* and *Dissolution* of Numbers.

13. Notation (as it is here considered) is,	1. *Certaine* and *Determinate.* 2. *Uncertaine*, *Undeterminate*, and *Arbitrary.*

14. *Notation* certaine and determinate, I here call that most excellent invention of expressing all Numbers whatsoever by these ten characters. *viz.* 1,2,3,4,5,6,7,8,9,0. whereof the first nine are commonly called figures, and the last a cypher; which signifies nothing, but only serves to augment a place.

15. These 10 *Characters* for the expressing of all *Numbers* are ordered into certaine places proceeding from the right hand towards the left in a *decimall progression*; so that the first place is of *Unites*, the second of *Tennes*, the third of *Hundreds*, the fourth of *Thousands*, &c. As here you may see in the Example following, where the order of the places is noted by the letters of the Romane Alphabet, and the value of them by the Capitall *Numerall* letters. X for the place of *Tennes*, C for the place of *Hundreds*, and M for the place of *Thousands*, &c.

t	s r q	p o n	m l k	i h g	f e d	c b a
1	2 3 4	5 6 7	8 9 0	1 2 3	4 5 6	7 8 9
m	m. m. m	m. m. m	m. m. m	m. m. m	m. m. m	C X
m	m. m. m	m. m. m	m. m. m	m m. m	C X	
m	m. m. m	m. m. m	m. m m	C X.		
m	m m. m	m. m. m	C X			
m	m. m. m	C X				
m	C X					Unites
38.						
	the sixth period is of thous: of thous: of thous: of thous of thous: of mill. of mill.	the fifth period is of thous: of thous: of thous: of thous: or mill of mill:	the 4th. period is of thous of thous: of thous: or thusãds ofmil.lõs	the third period of thousan: of thous; or millions	the 2d. period of thousands.	the first period of unites.

17 From hence all *Quantities* take their *Denomination.*

All quantities under the number of 10 are expressed by figures under (*a*) or in the unites place, all above 10 and under a 100, with figures under *b* in the tennes place, and with cyphers or figures in the place (*a*) that is tennes and unites, and so of the rest.

If the quantity consist of two figures, as of 57, it signifieth fifty seven, as if it should be 50 and 7. if of three figures as 789 it signifieth seven hundred eighty and nine, as if it should 700, 80, and 9. and after this manner

manner are all quantities valued that fall in the first period.

19. If the quantity fall in the second, third, &c. as consisting of more then three figures, note this alwayes, that after you have distinguished the figures into periods, as in the Table, the figure last in each period gives distinction or name to the rest in that period, as 75:221. signifieth 75 thousand; 221. & 325-317 signifieth 325 thousand, 317. Likewise 1:758-327: 821 signifie 1 thousand of Mill: 758 mill: 327 thousand, 821. that so having learned the value of three figures, the values of the rest are easily knowne.

21. Thus are all whole *Numbers* expressed in one ranke of figures. But *Fractions* are exprest in two rankes with a little line between them. The lower rule being called the *Denominator*, and the upper the *Numerator*; as if the 17 twentieth parts of any thing were to be expressed, it is thus done.

$$\frac{17}{20}\ \frac{\textit{Numerator.}}{\textit{Denominator.}}$$

20. An unite, or any whole thing may be conceived in the minde as divisible into any equall parts whatsoever, and these parts borrow their *Name* or *Denomination* from the number of parts supposed to be contained in that unite.

As if the unite be conceived to be divided into two parts, the parts are called seconds or halves, and the *Denominator* or *Name* of the parts will bee the figure 2. thus $\frac{-}{2}$. So if the unite bee supposed to bee

divided into three parts, the parts are called Thirds, and the Denominator 3, thus $\frac{}{3}$. If into 20 parts (as in the example before) then the parts are called twentieth parts, and the Denominator $\overline{20}$.

21. The number of parts signified in the Fraction is declared by the Numerator. For that part of the Fraction sheweth alwayes how many of those parts named are understood.

As if an unit were supposed to be divided into 3 parts, and one of those three parts were to bee exprest, it must thus be done, with the Numerator 1 over the Denominat. 3. *viz.* $\frac{1}{3}$ one third part, which of one shilling taken as the unite, and divided into 3 parts is 4 pence.

If two of these third parts were to bee exprest, it must be thus, with the Numerator 2 over the Denominator 3, *viz.* $\frac{2}{3}$, 2 third parts, which of a shilling taken as before is 8 d.

More Examples.

$\frac{1}{2}$ One halfe. $\frac{1}{5}$ one fift parts, $\frac{2}{5}$ parts, $\frac{3}{5}$ parts, $\frac{4}{5}$ four fift parts, $\frac{5}{7}$ five seventh parts, $\frac{1}{4}$ one fourth part, or one quarter.

$\frac{3}{4}$ Three fourth parts, or 3 quarters, $\frac{1}{6}$ one sixth part, which of a Crowne is 10 pence. $\frac{5}{6}$ five sixth parts which of a Crown is 50 d, or 4 sh. 2 d. $\frac{1}{7}$ one seventh part, $\frac{7}{10}$ seven tenth parts, $\frac{37}{100}$ thirty seven hundreth parts, $\frac{375}{4789}$ three hundred seventy and five,

foure

foure thousand, seven hundreth eighty nine parts.

Where in this last example, this unite is supposed to be divided into 4789 parts, and of these parts 375 are signified by the Fraction.

22. Thus are *Fractions* exprest at large, but wee use a more briefe expression of some kindes of them, by omitting the *Denominators*, when the parts are commonly known, and have names either Artificiall or Inartificiall, and that there be some marke or signe to distinguish them.

As because the twentieth parts of a pound are famously knowne by the name of shillings, we use to say and write 2 s. 3 s. 4 s. 5 s. 6 s. 7 s 8 s. 17 s. briefly with the Numerators only, and not fully with the Denominators, thus, $\frac{2}{20}$, $\frac{3}{20}$, $\frac{4}{20}$, $\frac{5}{20}$, $\frac{6}{20}$, $\frac{7}{20}$, $\frac{8}{20}$, $\frac{17}{20}$, of a pound.

The same wee doe also in the twelve parts of a shilling, saying, 1 d. 2 d. 3 d. 5 d. 10 d. briefly with the Numerators only, and not fully thus, $\frac{1}{12}$, $\frac{2}{12}$, $\frac{3}{12}$, $\frac{5}{12}$, $\frac{10}{12}$, of a shilling with the Denominator.

The like is said and exprest by the famous parts of weights, measures, time, &c. Weights are commonly Troy, or Averdepois either little or great.

Of *Troy* weight	a pound	is divided into	12 ounces
	an ounce		20 penny weight
	a peny weight		24 graines.

Of *Averde-pois* great weight.	112 pounds one quarter one pound	divided into	4 quarters. 28 pounds. 16 ounces.
Of *Averdepois* little weight	A pound An ounce	is divided into	16 ounces. 17 dragmes

All which are exprest in Account without Denominators by the signes,

As for	A pound, lb. A shilling, ss. A penny, d. A halfpenny, ob. An ounce, oz: or ℥.		A graine, gr. A scruple, ℈. A dragme, ʒ. A quarter, qr. A crown, Δ. A Ducket, duc.

A Karract Kr. &c.

23. From this *Abreviation* of *Fractions* ariseth that late most usefull invention of *Decimall Arithmetique*, where the Denominators are alwayes 1 with Cyphers, as 10, 100, 1000, 10000, &c. and therefore the Denominators are quite omitted (as alwayes certainly knowne being still more by one place then the figures of the Numerator) and the Numerators only set downe.

24. For distinguishing of the *Decimall Fractions* from the *Integers*, divers men have their divers waies. For some call their Tenth part Primes, the Hundreth parts Seconds, the 1000 parts Thirds, and marke them

them with equivalent Indices over their heads : as if they would expresse 245 whole, and $\frac{1234}{10000}$ parts of an unite, they would doe it thus, 245, 1′ 2″ 3‴ 4⁗. or thus, 245, 1 2 3, and they would read them thus; 245 whole, one prime, 2 seconds, 3 thirds, 4 fourths. Others doe nothing but set a point before the Decimall parts, thus, 245, 1234, others draw a line under them thus, 245, $\underline{123}$ writing the parts in smaller figures then the Integers. And thus much we intimate that the Reader may perceive the parts to be *Decimalls* upon sight.

But the best and most distinct way of distinguishing them is by a rectanguler line after the place of the unit, called *Seperatrix*, a seperating line, because it seperates the Decimall parts from the Integers. As in this example, where the Decimalls have their Denominations by 1 and cyphers, in a contrary order to the Integers, though in some of the worke following, a Comma is used instead thereof.

h	g	f	e	d	c	b	a	v		*a*	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
9	8	7	6	5	4	2	1	0		1	2	3	4	5	6	7	8	9
o	o	o	o	o	o	o	o	o	The unites place.	o	o	o	o	o	o	o	o	o
o	o	o	o	o	o	o	o				o	o	o	o	o	o	o	o
o	o	o	o	o	o	o						o	o	o	o	o	o	o
o	o	o	o	o	o								o	o	o	o	o	o
o	o	o	o	o										o	o	o	o	o
o	o	o	o												o	o	o	o
o	o	o														o	o	o
o	o																o	o
o																		o

25. Obſerve in this Table that there are four Progreſſions thoſe in the figures above, which are called Indices proceeding both wayes from unity are *Arithmeticall*, the difference from the unites place being 1. both progreſſions being the ſame from unity, though contrarily : the lower is a Geometricall continuall proportion from unity both wayes, every degree or place being ſucceſſive 10 times more then other.

As the *Series* of the *Numbers* from *Unity* are continued in a decuple proportion from the right hand towards the left, ſo their value do decreaſe in the ſame proportion from unity, as the other do increaſe above

bove unity; for let 3210,123 be a number given and stand with his letters thus,

b. equall to 10 a. a. equall to v. d b a v *a b c*
v. equall to 10|*a.*|*a.* equall to 10|*b.* 3 2 1 0|1 2 3
and |*b* equall to 10|*c.*

Againe,

a. equall 10. v.
a. equall $\frac{1}{10}$. v.
b. equall 100. v.
b. equall $\frac{1}{100}$. v.

27. Hence it follows that decimalls are set downe in a retrograde order to integers, for if you were to set downe in integers 7000 and 30, it would be thus, 7030.

But to set these downe in decimalls it would bee in a retrograde order, as if you were to expresse $\frac{7}{1000}$ and $\frac{3}{100}$ it would be thus |0307.

For the unite or one integer, is alwaies understood to be divided into parts, bearing the denomination or name of the place of the last figure in the decimall fraction.

As 0'1 signifieth one tenth part as if it were writ at large thus $\frac{1}{10}$ the denomination of the place being tenths, as the first place from the unite towards the right hand, and noted in the former example with (10)

So 0|12 signifieth 12 hundred parts, as if it were written at large thus $\frac{12}{100}$ the denomination of the last place being noted with (100) under the last figure 2.

0|123 signifieth 123 thousand parts, as if it
$\bar{0}$ were written at large thus, $\frac{123}{1000}$.
0
0

0|1234 are 1234 ten thousand parts, as if it
$\bar{0}$ were written at large thus $\frac{1234}{10000}$. and
0 so of the rest.
0

29. *Ciphers* before integers or after the *decimall* parts signifie nothing at all, but after the integers and before the decimall parts (that are next to the *seperatrix* on either side) they retaine their force, for the making up the places, whereby the values of the other figures are estimated.

As 0000001. signifieth but 1.

& 10000000. signifieth but $\frac{1}{10}$ part.

30. And therefore in writing of decimall parts, let the *seperatrix* be alwaies used, and let the void places (if there be any) be all filled up with Ciphers, and the place of the unite it selfe set downe though there be no integers.

31. The Index of unite is put to be 0. as in the former example, and the Index of any other place from unity is knowne thus, in Integers abate 1. from the number of places from unite, and the number remaining is the Index to be written downe in a greater figure as the Index of 3. in this number 73921. Is 3. of 7. is 4. of 2. is 1. taking their denomination from

from their diſtance in places from *Unity.*

In decimalls, the juſt number of decimalls from unity, is the Index to be written with a ſmall figure as the Index of 9. in this decimall 0|73921. is (3) of 1. is (5) of 7. is (1); Theſe Indices are of great uſe in the finding out the true values of numbers in Multiplication and Diviſion, as ſhall be after ſhewed.

The Learned Mr. *Oughtred* in his new *Clavis Limata* expreſſeth the *Indices*, of *Integers*, *Affirmatively*, and of decimalls *Negatively*, and ſo addes and ſubſtracts them as in Coſſick numbers, but becauſe it would a little trouble the new beginner to be told of the addition of ſignes, I mean of + and –; Out of that ground I ſhall after ſhew ſome few rules that with ſmall practiſe will performe the ſame.

32. The reaſon of theſe Indices (they increaſing in Arithmeticall proportion, as the numbers under increaſe in Geometricall proportion) is plaine out of Mr. *Oughtreds Clavis Limata*, Chap. VI.

Arithmeticall proportion working that in Addition and Subſtraction, which the other doth in Multiplication and Diviſion:

And you may obſerve, that in all other parts of Arithmeticke as in the Logar: Where by the learned advice of our Countriman Mr. *Briggs* the Log. of 1. or unite was put to be a Cipher, and the Log. of 10. to be 1. &c. As alſo in the Sexagenary account; where for degrees the Index is 0, and ſo to increaſe both wayes. Theſe Indices have the ſame worke,

and

and of the same use as in Decimalls.

I am plainer in this decimall way, for the great facility it brings with its practice in all the parts of Arithmeticke, Astronomy, and the rest of the Mathematicks.

In practicall Arithmeticke, if the first institution had been in Decrimalls, we had never been troubled with so many fractions; it were yet worthy the name of Reformation to cause the fractions of mony and weight to be altered: And as concerning the ease in measure, Surveyors and Land-meaters, who use the decimall chaine, and those who use a decimall foot, yard or scale can best certifie upon experience: It may be shortly (if other my occasions hinder not) I may publish some Tables, after the most easie and methodicall way hitherto thought on, towards the ready obtaining the true place of any of the planets in this decimall way; that will be a sufficient testimony of the ease thereof in Astronomicall calculations. Thus much for an Introduction into Decimall Arithmetick.

34. Hitherto hath been spoken of *Notation certaine*, and *determinate*, both for *Integers*, *Fractions*, and *decimalls*.

Now

Notation uncertaine, *undeterminate*, and *arbitrary* is when any quantity, either Magnitude or Number, whole, or broken, for the time present (or during the working of a question) is noted by any letter of the

the Alphabet, as by A.B.C.D.E.&c. And when the Queſtion is ended, and another in working, the ſame letters may ſignifie other magnitudes or numbers, according to the will of the *Arithmetician.*

35. The letters thus arbitrarily ſignifying magnitudes or numbers, are called their *Species*, and the ordering theſe in Arithmeticall forme is called in Latine *Arithmetica ſpecioſa* or *Logiſtica ſpecioſa*, and in Engliſh I call it, Arithmetick in *Species.*

36. The expreſſion of quantity or number in this way is moſt doctrinall and of great eaſe; for as in the ſecond part you ſhall finde that any Arithmeticall queſtion propounded, though otherwiſe very difficult, may eaſily receive ſuch a reſolution in *Species*, that thereby in numbers when you pleaſe it may bee anſwered; beſides the infinite helpe to memory in the invention of many rules therein.

37. As in Sect. 28. any quantity may be expreſſed by letters, as 48. whether pounds, yards, or miles, may be expreſſed by an A. or B. &c.

38. The ſigne of Addition or Affirmation is + (or pl. for *plus* more) as, + A, whereby is ſignified that any magnitude or number noted by A, is affirmed to be, and to be added, ſo + 34 more by 34.

39. The ſigne of Subſtraction or negation is − (or *m* for *minus* leſſe) as − B, whereby is ſignified that any magnitude or number noted by B, is denied to be, or to be wanted, or ſubſtracted, ſo − 34 leſſe by 34.

The

The ſigne alwayes belongeth to the magnitude or number following it.

And every Magnitude or Number which hath not the ſigne - prefixed before it, is underſtood to have the ſigne + although it be not expreſſed.

The ſigne of *Multiplication* (if any be uſed) is this ×, the difference of this ſigne from the ſigne of Addition is, that of Addition is St. *Georges* Croſſe, this is St *Andrews*, but moſt commonly letters are joyned together by Multiplication without a ſigne.

The ſigne of Equality is this =

Example. B+C=A to be read thus, B more by C, is equall to A.

So likewiſe B-C=E thus to be read, B leſſe by C, is equall to E.

So in Numbers ſuppoſing B to be 12, and C to be 3. then A will be 15. and E 9. thus,

$$12+3=15 \ \& \ 12-3=9.$$

Thus we have done with the Notation of Quantities both in Numbers and Species.

40. *Numeration* is that by which two numbers being given we doe rightly finde out the third, and is comprehended under *Compoſition* and *Diſſolution.*

41. *Compoſition* comprehends *Addition* and *Multiplication*; *Diſſolution*, *Subſtraction* and *Diviſion.*

Addition and *Subſtraction* being accounted the prime and ſimple parts of *Numeration*; *Multiplication*

- **Arithmetick** is either
  - *Simple*, which consisteth in
    - *Notation*,
      - *Certaine* and *Determinate*, and that more generally, as in certaine Characters, 1 2 3 4 0, &c. more specially in integers, as 1. 2. &c. or Fractions, as $\frac{1}{2}$. $\frac{2}{4}$. &c.
      - *Uncertaine* and *Arbitrary*, as by any Letters of the Alphabet, A, B, C, D, &c.
    - *Numeration* considered.
      - In the *composition* and that
        - Simply in *Addition* ——
        - Compounded in *Multiplication*
        - Which may bee called the *Genesis* of Numbers.
      - In the *dissolution* of it likewise,
        - Simply in *Substraction* —
        - Compounded in *Division*
        - Which may bee called the *Analysis* of Numbers.
  - *Comparative* in the *Ratio* of
    - *Quality*, Quantity, which is the simple consideration of the difference of two Numbers betwixt themselves.
    - which is the consideration of Numbers amongst themselves, and to our purpose is either,
      - *Arithmeticall* consisting in the same differences of Numbers
        - Continuall, as 2. 4. 6. 8. 10. the differences being equall.
        - Disjoyned, as 2. 4. 8. 10. the same difference being betwixt 2 and 4. 8 and 10.
      - *Geometricall* consisting in the like ha[illegible], or division of numbers, and is either
        - *Continued*, as 2. 4. 8. 16. &c. the like reason being betwixt them all.
        - *Discontinued*, as 2. 4 : : 8. 16. the like reason being betwixt 2 and 4. as 8 and 16 commonly called the Rule of Three, or the Golden Rule, which is either direct, reciprocall, or compounded.

Place this Table in the first Alphabet Fol. 17, at this Marke. ✠

tion, and Division the Conjunct; and now wee are to proceede to the rest of the worke;

But for the better remembrance of that which hath beene spoken, and the more easier understanding of that which hereafter is to be taught, it shall be necessary to give a *Breviate* of all in this little Table annexed, to be diligently observed, and imprinted in the memory of him that would desire with speed to be an understanding *Arithmetician*.

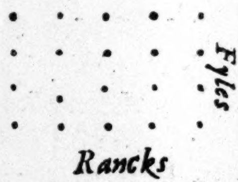
Thus having shewed (as it were) the summe of all the Arithmetician's taske, wee will proceede to declare the manner of operation in all the particulars mentioned, observing this order in this first part, *viz.* First, of the foure principall parts of Numeration; *viz.* Addition, Substration, Multiplication, and Division, in Integers, and Decimalls, with all such easie rules as may advantage the memory herein, then of Proportion. Lastly, the same foure parts againe, in fractions, and parts of numbers; and then to the second part in *Species*, and so by Gods assistance, passe through all the usefull parts of Arithmetick in order.

Chap. II.

Addition in Integers and Decimalls.

1. A*Ddition* is that part of *Numbring* or *Numeration*, whereby two or more numbers are added together, and ſo the totall or ſumme of them is formed.

2. For the eaſie apprehending of Addition, and Subſtraction, conſider theſe pricks ſtanding directly all in ſtraight lines, which wee call Rancks, and every prick one directly over another, which wee call Fyles.



3. In Addition of numbers, the numbers to be added are to be ſo orderly placed in Rancks and Fyles, as that all the figures of the ſame place, muſt ever ſtand in the ſame Fyle, and then a line is to be drawn under them.

4. Then beginning at the right hand Fyle, all the figures therein are to be added together, and their ſumme, (if it conſiſt but of one figure) is to be ſubſcribed under the line, and in the ſame Fyle, the ſame muſt be done in every other of the Fyles; but if the ſumme of the places or Fyle conſiſt of two figures, that

that is, amount above 9. then the right hand figure is only to be set downe, and the left hand figure to be reserved, and added to the next place or Fyle.

5. In Addition all the Numbers taken together are equall to the summe.

Examples.

(1)		(2)		(3)		(4)	
	42		9851		1646		981275
	56		7850		345		112345
	98		182		98		2981
			17883		1		32972
					2090		5498
							957
							82
							3981
							1140091

3

3

Explanation of the Examples.

In the first example it is desired that 42 and 56 may be added together, setting them as in the example in Ranck and Fyle, so that alwaies unites may stand under unites; wee say in the last Ranck, 6 and 2 makes 8, which is set under the line in that Fyle, then 5 and 4 makes 9, set under his owne Ranck.

In the second Example, it is desired to adde 9851, and 7850, and 182, in one summe, setting unites in

 one

CHAP. II.

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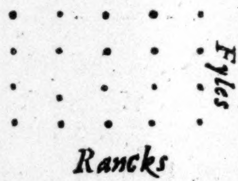


4. Then beginning at the right hand Fyle, all the figures therein are to be added together, and their ſumme, (if it conſiſt but of one figure) is to be ſubſcribed under the line, and in the ſame Fyle, the ſame muſt be done in every other of the Fyles; but if the ſumme of the places or Fyle conſiſt of two figures, that

that is, amount above 9. then the right hand figure is only to be ſet downe, and the left hand figure to be reſerved, and added to the next place or Fyle.

5. In Addition all the Numbers taken together are equall to the ſumme.

Examples.

(1)	(2)	(3)	(4)
42	9851	1646	981275
56	7850	345	112345
98	182	98	2981
	17883	1	32972
		2090	5498
			957
			82
			3981
			1140091

	3
3	

Explanation of the Examples.

In the firſt example it is deſired that 42 and 56 may be added together, ſetting them as in the example in Ranck and Fyle, ſo that alwaies unites may ſtand under unites; wee ſay in the laſt Ranck, 6 and 2 makes 8, which is ſet under the line in that Fyle, then 5 and 4 makes 9, ſet under his owne Ranck.

In the ſecond Example, it is deſired to adde 9851, and 7850, and 182, in one ſumme, ſetting unites in one

one Fyle, and the reſt as in the example. I ſay 2, and 0, and 1, makes 3. to be ſet in the ſame Fyle; then I ſay 8, and 5, and 5, makes 18. I ſet 8. in that Fyle, but carry the 1. to the next (which is to be obſerved in all) then I ſay 1. that I carried, and 1, and 8, and 8, makes 18. I ſet 8, in his Fyle, and carry 1. Laſtly 1. that I carryed, and 7, and 9, makes 17. both which are ſet downe in their proper places, as in the example.

The reſt of theſe examples are wrought as is ſet downe.

Addition in Decimalls.

There is little or no difference from the former, only alwaies ſetting unites in one fyle, then the ſeparating lines will follow directly one under another.

Examples.

(1)	3542‘1200 492‘9125 91‘7810 5189‘8921 9316‘7056	(2)	39‘0123 7‘51 918‘925 32‘7891 998‘2364

Nothing is obſervable in theſe Examples but that in the former the voide places are filled up with Ciphers, according to Chap. 1. Sect. 30. the latter is not: only

only in these as in the former, observe Ranck and Fyle, and march or proceed towards the right hand.

Addition of Fractions where the Demominators *are omitted.*

	l.	s.	d.
	135	17	8
(1)	95	11	2
	3	5	9
	234	14	7

	l.	s.	d.	ob.
	175	15	2	ob
	75	11	9	0
(2)	30	00	0	0
	71	11	11	ob
	131	10	10	0
	484	09	9	0

l.	oz.	d.	gr.
13	9	13	15
221	11	10	01
15	10	11	2
251	7	14	18

Observe alwaies to set figures of the same denomination in the same ranck, and then by Addition as before, summe up every particular row, and as many unites must you beare to the next denomination, as there be numbers of that denomination in that following: As in the first Example, I say 9. pence, and 2 d. and 8.d. makes 19. pence, in which I finde 1 shilling contained, and the Remainder 7. I set downe, and carry 1 shilling to shillings; then I say 1 shilling and 5. is 6. and 1. is 7. and 7. is 14. I set downe 4 shillings, and carry 10. then I say 10. and 10. is 20. and 10. is 30 shillings, that is 1.l. 10 s. I set downe 10.s and carry 1.l. which 1.l. is to be added to pounds, & pounds are alwaies added together as if they were whole numbers.

Now in the adding of pence together, for helping

of the memory, it is beſt to marke every ſhilling by a point, as in the ſecond example.

The third example being of Troy weight, there is no more difficulty then in the former, alwaies noting thus much to have a care to carry ſo many of the former denominations as are contained in that row you adde up, and ſet downe the remainer.

Chap. III.

Subſtraction in Integers and Decimalls.

1. S*ubſtraction* is that part of Numeration, where one number is ſubſtracted or taken out of another, and ſo the Remainder is gotten, which is alſo called the difference or exceſſe.

2. In ſubſtraction of numbers, the leſſer is to be ſet downe under the greater, as in Addition, orderly, obſerving to ſet unite under unite in ranck and fyle and a line is to be drawne under both.

3. Then beginning at the right hand, the particular difference of each place, are to be found by ſubſtracting the lower figure from the higher, and to be ſubſcribed in their proper places.

4. Bu

4. But if the higher figure be leſſer then the lower, 10. muſt be put to the higher figure, that the remainer may be taken and ſubſcribed, and then in the next place toward the left hand, either the higher figure is to be accounted 1 leſſe then it is, or elſe (which is more uſuall) the lower figure is to be accounted 1 more then it is, and then the ſubſtraction is to be made.

5. In Subſtraction, the number to be ſubſtracted together with the difference, are equall to the number from which the ſubſtraction is made.

Examples of Subſtraction.

(1)	58	(2)	456	(3(	4562	(4)	513217	(6)	1647
	32		321		1368		159832		1588
	26 *Rem.*		135		3194		353385		59

(5)

1000000
325782
674218

In the firſt example it is deſired to ſubſtract 32. from 58. I ſet 32. under 58. in ranck and fyle, then I ſay 2. from 8. reſts 6. and 3. from 5. reſt 2. which I ſet downe, as in the example; in the third example it is deſired to ſubſtract 1368. from 1562. the which after it is ſet downe is done thus, 8. from 2. I cannot, and therefore I borrow 10 to make it 12. and there remaines 4. to be ſet under the line in fyle, then I ſay one that I borrowed, and 6. is 7. which I cannot take

 from

from 6, but out of 16, there remaines 9. then one I borrowed, and 3, is 4, which from 5, reſts 1. And laſtly 1, from 4, reſts 3, as in the example.

If any numbers as in the fift example be to be ſubſtracted from 1, and Ciphers, as it ſometimes falls out in decimalls; but moſt commonly in the Logarithms, make the laſt figure equall to 10, ſetting down the remaine, and all the reſt make 9. as I make 2, 10, by ſetting 8, under, and all the other 9, by ſetting the remaine.

Of Decimalls.

5. There is no other difficulty in Decimals, obſerving to ſet unite under unite.

134‘5780	538275‘00019
12‘5812	4638‘0057
121‘9968	533636‘99449

6. If the decimall fractions have not an equall number of figures, the voide places are to be ſupplied with Ciphers eſpecially in the upper number, or to be underſtood to be ſo ſupplied.

Let 43‘6374, be ſubſtracted from 472.

Foure Ciphers are	472‘0000	Or to be	472
to be put to the	43‘6374	under-	43‘6374
greater, thus	428‘3626	ſtood thus	428‘3626

So

So in	65'2	Thus	65'200	532'64	Thus	532,64
these	48'375	fully	48'375	214	fully	214'00
	16'825		16'825	318'64		318'64

Substraction of Fractions without Denominators.

7. If the number out of which the Substraction is to be made, be in any of his Numerators too little for the number below it, you must borrow one of the former, and adde to it: as if in pence, you must borrow 1.s or 12, in shillings, 1.l. or 20.s &c.

	l.	s.	d.		l.	s.	d.	ob.
	375	11	5		1754	11	2	0
(1)	132	9	4	(2)	982	13	5	0
	243	2	1		771	17	9	0

As in the second example I am to take 5,d. from 14,d. because I borrow 12, d. rest 9, then 13,s. and 1 s. I borrowed makes 14 which from 31, rest 17, because I borrowed 1,l. or 20,s. then in pounds; 1, that I borrowed and 2, is 3, which from 4, rest 1, &c: and so on as in whole numbers.

And note heere, that it is better to take the lower from that you borrow, and adde the remainder to the number set downe, and write the summe in the remaine; thus to say 5 from 12 rest 7, with which 2 makes 9 to be set downe, then 14 from 20, rest 6, which with 11 makes 17 to be set downe, as above.

More

More Examples. *Of Troy weight.*

	l. s. d. ob.
Received (3)	7854-11-9-ob
Disbursed	321-15-5-0
	781-19-8-1
	983-00-1-0
	783 11 10-1
Summe	2870-7-1-0
Remaines	4984-4-8-ob
Proofe	7854-11-9-ob

(4)

l. oz. d. gr.
17—9-13—22
15-11-19—20
1—9-14—2

Of Time.

(5)

da. ho. min.
312-13-15
112-16—45
199-20—30

8. The proofes of Addition and Substraction are as followeth: in Addition cast away all the nines of your severall summes, and cast away the nines of your whole summe, and if the remainers be equall, you have done right, as in the second example of Addition you may see the proofe set downe to be 3. And note that in casting away 9, if two figures amount above 10, as if it be 13, carry 4 to the next, if 15, carry 6 to the next.

The proofe of Substraction is, that the lesser number and remaine must alwaies make up the higher number, which is soone done by Addition, as the proofe is set downe in the third example of Fractions undenominate.

9. And heere the yong Arithmetician may receive some

some comfort , for that he can now worke some questions of use by Addition and Substraction only; Example, one saith my Father was borne in the first yeare of *Henry* the 8, how long, or how old would he have beene in this yeare, 1648? I finde that *Henry* the eight began his Raigne in 1509, the which being substracted from the yeare present 1648, resteth 139, which is the time since , and would have beene the age of that man if he had lived.

And he may summe up the severall Pages of an Accompt booke in l. s and d. he may likewise tell the difference of any two numbers, as in the last example of plaine numbers ; how long since 1588, the yeare that the Spaniard thought to have invaded England, which being substracted from 1648 , rests 60.

In the third example of *Fractions* undenominate, if 7854, l. 11, s. 9, d. ob. were given in stock , you have disbursed at severall times as appeares 2870, l. 7, s. 1, d. which substracted from the Principall rest 4984, l. 4 ,s. 8, d. ob. yet remaining to account for.

9. Of *Bypartition* ; I must heere a little digresse from the order, and teach how to take the halfe of any Number, or to divide a Number by two , which is thus: if the Number consist of even figures take the halfe thereof, and set it under the Number ; but if it consist of uneven Numbers , if you begin from the left hand , taking the halfe of it, augment the figure following by 10, if the figure before were odde and so go on.

Ex. 1.

Ex. 1. $\frac{4862}{2431}$ (2) $\frac{47892}{23946}$ (3) $\frac{9011780}{4505890}$ (4) $\frac{2350}{1175}$

In the first example set downe only the halfe of each number, in the second, say the halfe of 4 is 2, of 7 is 3, of 18 is 9, of 9 is 4, of 12 is 6, and I tearmed 8, 18, because 7 was an odde number.

10. The halfing of any number from the left hand was shewed by the last, but it will be of more speed in the workes following, to doe it from the right hand, which is only by observing that if the figure standing in ranck next before be odde, you must account the figure next after 10, more then it is, as in the second example, beginning at the right hand with 2: I say the halfe of 12, (because 9 is odde) is 6, the halfe of 9 is 4, the halfe of 18, (because 7 is odde) is 9, the halfe of 7 is 3, the halfe of 4 is 2.

Chap. IIII.

Addition and Substraction of Indices.

1. FOR Addition if they be Homogeneall, or alike, that is both of Integers or Fractions: adde them together, the summe will be the Index of the same kinde; but if they be of divers kinds, take the difference

difference of them, which shall be of the same kind of that figure which was greater, or where the excesse lay.

Ex. (1)	3	(5)	7	3	(3)
	4	(2)	(2)	(2)	7
	7	(7)	5	(5)	4

In the first and second Examples I adde up the Indices, keeping the nature and kinde, in the 2, d. and 3, d. I take the difference setting it downe to the kinde of the greater figure.

2. For Substraction, placing them one over the other, (and it matters not whether be greater) then is there two varieties, and of each variety two cases: for the first they may be both Indices of Integers or Decimalls, and the higher figure is either greater or lesser then the lower, or secondly they may be of divers kindes with the same variety. Now if they be both of Integers, take the difference, which if the higher figure be greater, the remaine is an Integer: if lesser a decimall: but if both be of decimalls, taking the difference, it shall be of decimalls if the higher be greater; of Integers, if lesser.

5	4	(5)	(4)
4	5	(4)	(5)
1	(1)	(1)	1

In the first two, they being Indices of integers, the remaine of the former is of Integers, of the latter of Decimalls, because of the difference of the higher figures, and so of the latter observing the Rule.

If

If the Indices be unlike, that is, the one of Integers, the other of Fractions, ſetting them orderly, the one under the other, take the ſumme of them, (making it alwaies of the ſame kinde that the higher Index is) that is, if it be of an Integer, of the ſame kinde, if of a Decimall, a Decimall.

5	(5)	7	(3)	4	(4)	2
(2)	2	0	0	(4)	4	(5)
7	(7)	7	(3)	8	(8)	7

In theſe Examples, you take the ſumme (the kind being the ſame with the higher figure) as in the firſt 5 and 2 is 7, which is of the ſame kinde with 5, the Index of an Integer. This is to be well practiſed (being of great uſe, as well in Decimalls, and Logarithmes, as in Logiſtica Sexag.) and in Subſtraction have a care to place your numbers as directed, it being no matter whether of them be greater.

Chap. V.

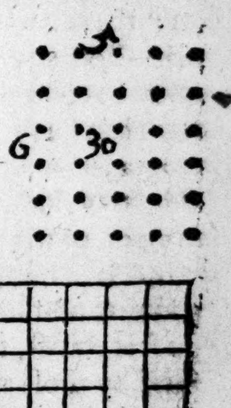
Of Multiplication.

1. *Multiplication*, is a part of conjunct Numeration, or numbring, whereby the *Multiplicand* (which is the number to be multiplied) is ſo often added

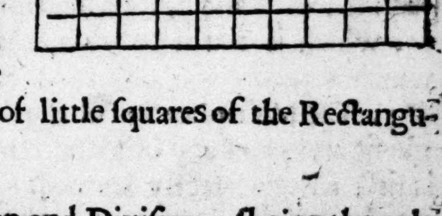
added to it selfe, as an unite is contained in the Multiplyer (which is the number multiplying) and so the *Factus* (or Product) which is the result of the worke, is had.

2. The number thus found, is called the Product and *Factus*, that is, the number made, and then the numbers to be multiplyed, are called the *Factores*, Makers. This number, is also called the Rectangle, or the Plaine, and then the one of the numbers is taken for the length, and the other for the breadth of a Rectangular Plaine, as heere,

In unites 6 the length, and 5 the breadth; the Plaine being 30.



Or in little squares made of the measure (such a foote, or the like) 10 being the length in such measures, and 5 the breadth, the Product being 50 of little squares of the Rectangular Plaine.



3. Multiplication and Division, (being the only *Remora's* that perswade those (who delight to take

small

ſmall journies, and little paines) from the knowledge of this ſweete and pleaſant Countrey of Arithmetick, conceiving it *terra incognita*, I ſhall endeavour to render the eaſieſt wayes that have yet beene found out to performe the ſame. If the Logarithmes could worke numbers to ſome few Periods, as to three, then I muſt confeſſe great labour might be ſaved; in the meaue time, I ſhall expreſſe two waies: the one for multiplying all Numbers by Duplication, Triplication, Reduplication, and Bipartition; the other by that laudable Invention called Napiers-Bones; with ſome novelties therein, as by them eaſily to contract, or cut off as many figures to your right hand, as you pleaſe (the ſame being done by Retrogradation, invented and plainly ſet downe by Mr. *Oughtred*, in his *Clavis Mathem*) as alſo by the old manner, of the which three the Reader may chuſe.

4. If a number be compounded of two numbers, and that number multiply another number; the Product is equall, to the Product of that number multiplied by thoſe two numbers. Examp. *6*, is compounded of 2 and 3, let *6* multiply *9*, the Product is 54, which is equall to *9* × 2, that is 18, and 18 × 3 which is 54.

5. The Multiplication of the ſingle figures by ſingle figures, or of any other number, by ſingle figures, is firſt to be perfectly learned. The Multiplicand is to be written downe, and the Multiplyer under it, which afterwards is taught.

Duplication,

Duplication, or Multiplying by two.

6. If any Figure, or Number, be to be multiplyed by two, beginning at the right hand, call every figure by a name double to it selfe, and set it downe, if the figure exceede 4, you must account the next on the left hand one more then the double.

34231		3987217		590019
2	(2)	2	(3)	2
68462		7974434		1180038

In the first (in my minde) I double each figure, saying, 2, 6, 4, 8, 6, and set them downe, and note this, that you must not use to say 2 times 1 is 2, and 2 times 3 is 6, &c. nor to say 1 and 1 is 2, and 3 and 3 is 6, but (for the reasons following) double each figure in your minde.

In the second, I say 14 or 4, going one in minde, and 3 (because I carryed 1) and 4, and 4, and 7, (because I borrowed 1) &c. and heereof (as also of Bipartition) the Reader must be ready, there being no difficulty at all in doing it, and much use, as followes.

Triplication, or Multiplying by 3.

7. Adde the double 5 to the Number given, the Summe is the Product.

Example.

24321	793256
3	3
72963	2379768

Heere you still carry the double in minde (it being perfectly learned) as in the first, I say 2 and 1 is 3, 4 and 2 is 6, six and 3 is 9, 8 and 4 is 12, 4 and 2 is 6, and 1 is 7, all which are set downe.

In the second, I say 12 and 6 is 18, ten and 5 and 1 is 16, foure and 2 and 1 is 7, six and 3 is 9, eighteen and 9 is 27, fourteene and 7, and 2 is 23, all to be set downe.

Reduplication, or Multiplying by 4.

8. This is doubling the Duplication in your minde.

Examp.

2345	549276	5001210
4	4	4
9380	2197104	20004840

In the first I say 10 and 10 makes 20, I set downe a Cypher and carry 2 in my minde, then 8 and 8 and 2 makes

makes 18, setting downe 8 and carry 1, then 1 and 6, and 6 makes 13, and 1 and 4 and 4 makes 9, and so of the rest, as you may see in the other Examples, which will be very easie, the greatest number (to keepe in memory) being but 3.

To Multiply by 5, 6, or 7.

9. First you must know, that to multiply any number by 10, is, but to put a Cypher to the end thereof; therefore to multiply any number by 5, (having put, or conceived to be put a Cypher to it) take halfe that number, and you have multiplyed it by 5 as to multiply 57831 by 5, having put a Cypher to it, thus 57831|0, take halfe, thus the halfe of 10 is 5, of 11 is 5, of 3 is 1, &c. 289155 is the Product.

	31251 3|0	But it is best to conceive the Cypher annexed, thus	5298001
So	5		5
	1562565		26490005

To multiply by 6, is to take halfe, adding a Cypher, and to adde to the halfe the figure standing next before.

2132|0	34578|0	9825458|0
6	6	6
12792	207468	58952748

In the first, I say the halfe of 0 is 0, and 2 is 2, the halfe of 12 is 6 and 3 is 9, the halfe of 13 is 6 and 1 is 7, the halfe of 1 is 0, and 2 is 2, the halfe of 2 is 1, all which are to be set downe in the Product, and so of the rest.

To multiply by 7, is to take halfe, and adde it to the double of the former figure, still conceiving a Cypher to be added as before.

2132|0	34578|0	9125458|0
7	7	7
14924	242046	63878206

In the first, I say the halfe of 0 is 0, aud 4 is 4, the ½ of 12 is 6, and 6 is 12, I set downe 2 and beare 1, the halfe of 13 is 6 and 1 is 7 and 2 is 9, the halfe of 1 is 0 and 4 is 4, the halfe of 2 is 1, and so of the rest.

To Multiply any Number by 9, or 8.

10. First conceive the number multiplyed by 10, by putting to a Cypher, and then substract each former figure from the following, beginning with that next before the Cypher, and the Remaine is the Product of that number multiplyed by 9.

Example.

Example.

3785|0	39801210|0	525291|0
9	9	8
34065	258210890	4727619

In the first I substract 5 from 10, remaines 5, 9 from 15. remaines 6, 8 from 8, remaines 0, 3 from 7, remaines 4, and 0 from 3, remaines 3.

To multiply by 8, is to double each former figure, and substract it from the following, as before.

Example.

3785|0	39801210|0	5123012|0
8	8	8
30280	318410680	40984096

In the first, I double the former, and say 0 from 0, remaines 0, then 17, or 7 from 15, remaines 8, then 15, or 5 and 1 (I borrowed) from 8. remaines 2. then 6 and 1 (I borrowed) from 7, remaines 0, and 0 from 3, remaines 3, and so of the rest.

Thus hath been shewed Multiplication by one figure, with no trouble at all, only by multiplying by 8 and 7, there is a little, which by a little practice, and the due observation of the Multiplyer may divers times be cleared.

11. If the Multiplyer consist of more figures then one,

one, their ſeverall Multiplications are to be made of all the figures of the Multiplicand, by every figure of the Multiplyer, particularly, as was ſhewed before of one, and their ſeverall Products are to be written downe a skew, ever beginning at the place of the figure multiplying, and ſetting that particular Product alwaies under that figure.

This being done, all thoſe particular Products, muſt be added together into one ſumme, which will be the *Factus*, or totall Product of the whole Multiplication.

12. There may be many times a totall Product given at once of 2, or more figures; for if you adde two Cyphers to any ſumme, and take the fourth part, it is multiplyed by 25. &c.

13. If Decimall parts be to be multiplyed either by themſelves, or mixed with Integers; there is no difference at all from the Operation before declared; but only in this, that when all the worke is ended, there muſt be cut off with the *Severatrix*, as many places of the generall *Factus*, or Product, as there are places of Decimall Parts in both the *Factores*, for juſt ſo many alſo there muſt be in the *Factus*.

14. For the ſpeedy working in Multiplication, obſerve this Rule; that every ſeverall Product may containe one place more of figures, then there is in the Multiplicand, either as its place, or by prefixing a Cypher before the firſt figure, as in the Examples following will appeare, and by this meanes you ſhall

keepe

keepe a correſpondent habit, in your ſeverall Products, and ſo to multiply by what figure of the Multiplyer you pleaſe, which will be of great eaſe in this way.

Example.

32569	9657	9657
527	4876	4876
227983	57942	38628
065138	67599	77256
162845	77256	67599
17163863	38628	57942
	47087532	47087532

In this Multiplication, where the Multiplyer is of more figures then one, you may finde much eaſe in obſerving in the figures to make up the Products. As in the firſt example, I would multiply firſt by 2 (leaving ſpace for the Product of 7) then by 5, and adde thoſe 2 Products, they make up the firſt Product of 7, or which is all one, ſet them downe as you pleaſe, keeping them in their due places.

So likewiſe in the ſecond example I would firſt multiply by 4 by reduplication, then by 8 by doubbling the former Product, then by 6. which Product added to the Multiplicand, you have the Product of 7.

238'645
821'75
0715935
1670515
0238645
0477290
1919160
197101'75585

0'7538
0'5689
67842
60304
45228
37690
0'42883682

In the former Product 5 decimalls are cut off according to the quantity in both Multiplicand and Multiplyer, the like in the second the number being 8.

15. If one or both of the numbers propounded to be multiplyed have Cyphers annexed at the right hand, the Multiplication may be made of the other significant figures, as before, and then at the last so many Cyphers are to be added to the right hand, as the number of them amounted to.

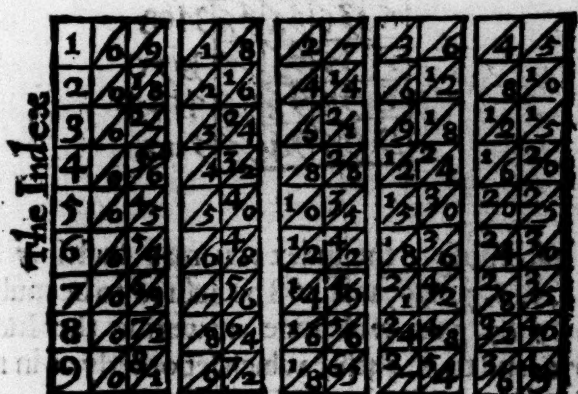
5500
250
275
110
1375000

63542'57
64000
25417028
38125542
406672448000

Of Napiers-Bones.

19. The Rods are best made of Wood or Ivory foure square, having all the digits on them and their Multiplication to 9, being only *Pythagoras* Table cut into peeces; they have an Index prefixed shewing the values of the Multiplees to 9. The Complement, *viz.* remainder to 9 is on the back side of each bone, the other sides being disposed to the most convenient forme, the figures represented being set on the ends or bottome.

The Picture of 5 sides, and there remaines to 9.



17. Having any Number given, you may tabulate or place it on the bone, observing that if you see not the

the figure on the foreside directly; yet if you see the remaine of it to 9. it is on the other side, which you may multiply by any figure as your Index directs, taking the sum of every Diagonall square, and setting them downe from the right hand, or after some practice from the left hand.

According to the Index you may multiply by 2, 3, 4, 5, 6, 7, 8, 9, as if 568 be desired to be multiplyed by 67, you will finde answering the Index 7. 3976. and to 6. 3408, which set orderly as in multiplication, will give the Product 38056.

18. In Multiplication set alwaies the Multiplicand on the Bones, and take by helpe of the Index every severall product answering the figures of your Multiplyer,

Multiplyer, which all added together make up the Product.

19. Thus may Multiplication be wrought at large; but oftentimes it falls out, especially in Decimall parts, that there shall be no neede to expresse all the figures of the Product; but only some of them towards the left hand, because those towards the right hand are of small value, and in many operations not considerable, which by the Bones may most excellently be abbreviated thus.

Consider what places you will keepe of Decimalls or how many you will cut off towards the right hand, as in this Example, 5'683 being multiplyed by 4'329, I finde there will be 6 places of Decimalls, but 3 will be sufficient for my purpose, therefore I would expresse no more worke then for 3 Decimalls, setting 5683 on the Bones: Let there be first a little distance or seperation betwixt 5 and 6, and now multiplying by 9, take off the Bones 51 which answereth on the 5 bone, having respect to what riseth in the tens place, on the other bone as you may see by the Example, which set close to the line, then removing the next bone, make the seperation betwixt 6 and 8, and take off answering 2, the Number 114, which set close to the line, and so removing the bones one further till the worke be ended, as in this example.

The

5	'683
4	'329
51	...
114	..
1705	.
22732	
24601	

The like may be practised in Numbers without the bones, observing the Rule above, and every time to take in one figure more.

Another Example of this contraction, where it is desired, that the Product may be pure, and all Integers.

	0568'3291
	3'2851
0	....
03	....
045	.....
0114	....
01705	...
1867	

In this Example I put a Cypher before the Multiplicand to make so many places as must be cut off, *viz.* and the first bone was seperated from the Index, *viz.* the Cypher, and then another added, &c.

20. I have shewed two easie wayes of practice in Multiplication; now it remaines to set downe the

usuall

usuall way ; so that the Reader may please himselfe, wherein the Multiplication of the single figures among themselves, is first perfectly to be learned by heart.

As 2 times 2, is 4. 2 times 3 is 6. 2 times 4 is 8. &c. 3 times 3 is 9. 3 times 4 is 12. &c. as in this Table is expressed, wherein you may enter with your two Figures, the one above the other on the side, and the square answering to thē both in the cōmō angle, containes the Product of these two numbers.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	26	45	54	63	72	81

21. If two or more figures be to be multiplied by one, the Multiplicand is to be writtē down, together with the Multiplyer as before, and then every particular figure of the Muliplicand in order from the right hand toward the left, is to be multiplyed by that one figure of the Multiplyer, and the particu-

lar

lar Products are to be set downe in their places, if they be written with one figure; but if any be written with two figures, the right hand figure only is to be set downe, and the left hand figure to be reserved as in Addition, and added to the next particular Product, as for example.

2132	31461
3	6
6396	188766

In the latter I say 6 times 1, is 6. which I set downe, then 6 times 6, is 36. I set 6 downe and beare 3 in minde, then 6 times 4. is 24, and 3 is 27. I set 7 downe and beare 2. and say 6 times 1 is 6, and 2 is 8. which is set downe, and lastly say 6 times 3 is 18, which is also set downe.

22. If the Multiplyer consist of more figures then one, you must worke as in the last Section by one figure, and then place them as is taught in Section 11 hereof, examples are there to be seene.

23. Because shillings are turned into pence by multiplying by 12, I will now before I make an end of multiplication, shew how it may be done all at one worke.

Double the number of shillings, and adde the double of each figure to the figure following, remember to adde a Cypher to the last place.

3452|9	7852337|0
12	12
41424	94227840

24. Th

24. The true place in the Product of the Multiplication of any two numbers together, is gotten by adding their Indices together, according to Sect 2. Chap. 4. as in this Example, 51'231 * by '3123) * I would know if 2 in the Multiplyer be multiplyed by 5 in the Multiplycand of what place, or how far it is distant from unity, and whether a Decimall or Integer: The Index of 5 is 1. of 2 is (3) I finde by the worke it is a Decimall, and that it is two places distant from unity, and so of any other.

$$\left\{\begin{array}{c} 1 \\ \underline{(3)} \\ (2) \end{array}\right.$$

Of Multiplication of Sexagesima's, and Sexagena's.

Sexagena's are the collection of degrees, all under 60 standing in the place of unite, and having a Cypher to his Index, then by collection all above 60, and under 360 fall in *Sexagena primis, &c.*

Againe a degree is divided into 60. one minute into 60 their Indices thus marked.

Sexagena's						*Sexagesima's*				
&c.	4.	3.	2.	1.	0.	(1)	(2)	(3)	(4)	&c.
As thus.	⁗.	‴.	″.	′.	o.	′.	″.	‴.	⁗.	

To Multiply these and to finde of what denomination the Product is off, is by adding their Indices as before is taught

3	$32'''$ multiplyed by $17'$ makes
(1)	$17'$
2	$544''''$ that is $9'''$ and $4''''$.

To multiply $5^{\backprime}\ 13^{\circ}\ 17'\ 18''$,
by 10

$50^{\circ}.\ 130'.\ 170''\ 180'''$
that is $52^{\circ}.\ 12'.\ 53''$.

And so of what kinde soever they be of, their denominations are easily found.

This have I, with as much plainenesse and brevity as possibly I could run over the practise of Multiplycation, whose effects are so many, that of necessity he that intends to understand any thing in Arithmetick, must be cunning and well practised therein, for by it, is the content of any *superficies* knowne, whose length and breadth is given, by the content and sum of any thing, the values of many of that kinde are found, and generally usefull in Planimetry, the Golden Rule, and in all the parts of Arithmetick; so now wee will proceede to the practice of Division.

Chap. VI.

Of Division.

1. DIvision is that part of conjunct *Numeration*, wherby one Number is substracted from another, as often as it is contained in it; and by that meanes it is found how many of the one is contained in the other.

2. The Number to be divided is called the *Dividend*; and the Number dividing is called the *Divisor*; but the Number found is called the *Quotient*, because it sheweth *Quoties*, how often the Divisor is contained in the Dividend. The Quotient is also called *Parabola*, because it ariseth from the comparing or Application of a plaine Number to the length given, that the breadth thereof may be found: As if the plaine Number 20 (described in plaine by his unites, 2 Sect. Chap. V.) be applyed to his length 5, his breadth will be found by Application or measure (as it were) to be 4; (and if a Number be applyed to another Number, with a little line betweene them as $\frac{20}{5}$ or $\frac{5}{13}$ it is thereby understood that the higher

Number is to be divided by the lower to which it is applyed, which is the forme and nature of all *Fractions.*

Wee will first shew the worke of Division, which will seeme at first somewhat difficult, and afterwards declare the easie way by Nepaires bones, or by Numbers made in their forme and nature. First, in Division wee begin at the left hand, and after that wee have distinguished out of the Figures of the Dividend, a Dividuall sufficient for the Divisor; wee try how often some of the first Figures of the Divisor may be had in the first Figures of the Dividuall, and it being knowne how often wee set in the place of the Quotient a Figure signifying how often, as 1 for once, 2 for twice, &c. then the whole Divisor must be multiplyed by the particular Quotient found, and the *Factus* must be substracted from the dividuall, and the remainder set under a lyne: Then the next Figure of the Dividend is to be set with the remainder; and another particular Quotient to be found, as the first was found, and the Multiplication and Substraction to be made as before, the same worke must still be repeated till wee come to the end of all the Dividend: But every particular Quotient thus found out, ought to be of the same degree, or place with that Figure of the Dividend which standeth (or is supposed to stand) above the unites place of the Divisor, in every particular Operation.

4. If after the joyning of a Figure of the Dividend

dend with the former remainer, and another figure of the Dividend to be put to the Dividuall, and this to be repeated as oft as neede shall require. And the remayner at last (if there be any) is usually set over the Divisor with a little lyne betweene them in forme of a Fraction.

5. If the Divisor hath Cyphers at the right hand they may be omitted, and so many of the last figures of the Dividend cut off, and the Division made in the rest of the figures; but after the Division is made the Cyphers are to be restored to the Divisor, (and the figures cut off) to the remainer, and the remayner and Dividend set in forme of a Fraction.

6. But if instead of the fraction so made with the Dividend and remayner, you desire to have Decimall parts annexed to the end of the Integers of the Quotient, that may expresse that fraction; you must continue on the Division supplying the voyd places of the Dividend with Cyphers, cut off with the *Seperatrix* in forme of a decimall fraction; or where figures are cut off from the Dividend for the Cyphers of the Divisor, you may continue on the Division through them, being cut off as decimal parts, and if these be not sufficient you may supply the defects with Cyphers.

7. In Division, as the Dividend is in proportion to the Divisor, so the Quotient is to an unite; as if 24 be divided by 6, the Quotient will be 4, therefore as 24. 6 :: 4. 1.

8. If a Quantity either Magnitude or Mumber, be made of two other Quantities by their Multiply-cation, the one of them being the one *Factor* will measure (that is evenly divide) the same *Quantity* made, by the other (being the other *Factor*.) As the Quantity 20 being made of 5 by 4, the one of them 5 will measure (or evenly divide) 20 by the other 4: so likewise 4 will measure the same 20 by 5. For the measure of 5 is 4 times, and the measure of 4 is 5 times in 20.

9. If a Quantity either Magnitude or Number, be made of two other Quantities by their Multiplication, it is all one whether you divide any other number by that one Quantity, or by those other two Quantities, that makes up that Quantity by Multion: As if 24 be to be devided by 6, the Quotient is 4: so if you devide the said 24, first by 2, the Quotient is 12, and againe that 12 by 3, the Quotient is 4, equall to the Quotient when the Divisor was 6, as before, for 3 multiplyed in 2, produceth 6: in the same manner, it is all one whether you devide by 12 or by 4 and 3. by 16. or 4 and 4. by 24, or 6 and 4. by 24, or by 3 and 2, and 4, for $3 \times 2 \times 4 = 24$.

10. *Bipartition* was shewed before; *Tripartition* or deviding any number by 3 is done easily without any Substraction, for beginning at the left hand, take the third part of each figure, and what remaines accounting 10 if one remaine, or 20 if 2, to the next worke as in this example; take the third part

part of 965427, the Quotient is 321809. saying the 3d. part of 9 is 3. of 6 is 2. of 1 (rests 1) of 24 is 8. of 2 is 0 (rests 2) of 27 is 9. this with a little practise will be found easie.

Likewise to devide any number by 4, is to take the fourth part thus, if 1789012. were to be devided by 4, the Quotient will be 447253. saying the fourth part of 17 is 4. of 18 is 4. of 29 is 7. of 10 is 2. of 21 is 5. of 12 is 3. likewise the fourth part of 32598763 is 8149940 $\frac{3}{4}$.

Likewise to devide any number by 5, is to cut off the last figure with the *Seperatrix*, and double all the former figures, only if the last figure be either 5, or a figure above it, you must set the excesse of it above 5, with 5 in manner of a Fraction, and adde one to the double of the figure going before; as if 5, devide 35785|4 the Quotient is 71570 $\frac{4}{5}$ or in a Decimall which is alwaies better to avoyde Fractions, 71570'8, by doubling the remainer: so if 5 devide 3278902|7 the Quotient is 6557805'4.

To divide any number by 6, is to devide that number by 3 and 2, as to devide 3568 by 6 thus

6) 3568 (594'666 according to the rules afore going.

```
30
 56
 54
  28     -4|0
  24 -    40
```

So 8 may be devided by 2 and 4.
9 by 3 and 3 } 16 by 4 and 4
12 by 3 and 4 } 24 by 6 and 4, or 2.3 and 4.

Examples of Division in Integers and Decimall Fractions.

```
                                 Dividend   Quotient
                                 54789'00 (6848'62 &c.
     (1) Divisor 8)              48
                                 --
                                  67
                                  64
                                  --
                                   38
                                   32
                                   --
                                    69
                                    64
                                    --
                                     50
                                     48
                                     --
   (2)       340|970 (56 4970/6000  20
   6000)     30 |000                16
             --                     --
              40 or in Decimalls.   40 &c.
              36
              --   (56'828 &c
              4'9
              48
              --
              '17
              12
              --
               50
               48
               --
               20 &c.
```

Dividend

Dividend Quotient

Divisor 385) 7892345 (20499 $\frac{230}{385}$
770....
(3) 1923 or in Decimalls
1540 (20499'597 &c.
3834
3465
3695
3465
2300
1925
37'50
3465
2850
2695
1550 &c.

In the first example according to the rule I enquire how many times 8 in 54, the which I finde 6 times, 6 I set in my Quotient and multiply it by 8, my Divisor which makes 48, the which I take from 54 remaines 6, the which I set under the line and place the next figure in my Dividend 7 by it, and worke till the end after the same manner.

In the second I am to devide 340972 by 6000 I place my Cyphers to the latter end of my Dividend and devide by 6, the Quotient is 56 $\frac{4972}{6000}$ or in working according to the directions in

Sect.6 it is in Decimalls 56*828 &c.

In the third having placed the Dividend and Divisor, I enquire how many times 3 is contained in 7 (having alwaies regard to the quallity of the figure following, being multiplyed by the last figure in the Quotient, that the Product may be substracted) I finde it twice (saying in my minde twice 3 is 6, which taken from 7 rests 1, which with the next figure following makes 18, and twice 8 is 16, which will come out of 18) and set downe 2 in the Quotient, then multiply the Divisor by 2 the Product is 770, which I substract from 789 rest 19, to which I adde the next figure of the Dividend 2, and finde it lesse then 385 my Divisor; therefore I put a Cypher in my Quotient, and fetch another figure 3 from the Dividend to make my number bigger then the Divisor, and then worke as in the example.

But because in chosing out of due Quotients, there is some trouble, the danger either to take it too great or too little, whosoever shall make use of Nepaires-bones shall finde much ease in them, and great speed.

Division by Nepaires-bones.

11. Set the Divisor on the top of the bones as was taught before in Multiplication, then have you that Divisor multiplyed by all the figures to 9, out of which you may chuse such convenient Divisors, as

as will be next lesse to the figures in the Dividend, and subscribe the Index answering in the Quotient, and so continually worke as before is taught. As for Example, I were to divide 34789021 by 568.

1	/5	/6	/8	568
2	1/0	1/2	1/6	1136
3	1/5	1/8	2/4	1704
4	2/0	2/4	3/2	2272
5	2/5	3/0	4/0	2840
6	3/0	3/6	4/8	3408
7	3/5	4/2	5/6	3976
8	4/0	4/8	6/4	4544
9	4/5	5/4	7/2	5112

568 34789021 (61248 $\frac{177}{568}$
3408
0709 or in Decimalls
0568 61248'31 &c.
1410
1136
2742
2272
4701
4524
177'0 6'60
1704 568
920 &c.

In

In this example I ſet 568 on the bones; which gives me the ſeverall Multiplees, ſet (for the better underſtanding) cloſe by the bones, then I conſider my Dividend, and what figures of the begining of it I muſt have, and I finde 4 *viz.* 3478, becauſe 347 is leſſe then my Deviſor, then I look on my bones (for the figures ſet by them are but for illuſtration) for a number that will be next leſſe then 3478: the which I finde to be 3408; which I ſubſcribe under the other, and draw a line, puting the figure *6* in the Quotient being found anſwering on the Index, the number 3401: then I make my ſubſtraction as before, and worke out my Dividend, finding the Quotient to be *61248* $\frac{177}{568}$ or in Decimalls 61248'31 &c. But a little practiſe will make this as plaine as can be.

12. You may likewiſe, which will be a great eaſe, eſpecially to thoſe that are not well practiſed in Diviſion, multiply the Diviſor (by doubling, trebling, redoubling, halfeing &c.) inſtantly to 9, which will ſerve you inſtead of Multiplees to be taken of the bones. Example, I am to devide 71234568, by 487.)

7) 487

```
7) 487|1   71234568 (146272'213 &c.
   974|2   2253
  1461|3    3054
  1948|4     1325
  2435|5      3516
  2922|6       1078
  2409|7        1040
  3896|8         0660
  4383|9          1730 &c.
```

The Divisor may be prepared of some of the Multiplees and yet you may have the rest as you please thus, and the intermediate places may be supplyed by memory if need be.

487	1
974	2
1948	4
3896	8

In this example I set down my Divisor, doubles it, trebles it, doubles the double, takes halfe, doubles the treble &c. and sets down the Indices 1.2.3.4. &c. answering every severall Multiplication, then considering my Dividend, I finde it will be once out of 712: and heere having the Multiplees already set downe, I substract the Divisor from 712 rests 225, to which I adde 3, and then looke the next lesse which answereth the Index 4, I set 4 in the Quotient, and substract 1948 from 2253 rests 305, I worke on &c. and this will be found as short a worke as needes be, for

for though I multiply my Divisor ; yet I save subscribing all the severall products under each part of the Dividend.

Thus have I shewed you how to devide all numbers ; but sometimes you are to divide a number by a Decimall of many figures , for the which I shall shew you this Contraction following by the bones.

13. If the Divisor have figures sufficient , you may continue the Division by diminishing the Divisor of a figure at the right hand, continually at every operation , which by the bones is done thus, consider what places you will have in the Quotient, and augment the Dividend with Cyphers accordingly , then if the Divisor be greater you must cancell so many figures as exceede , then at every operation you must take one bone off from the right hand, and seperate it a little for distinction sake , having consideration to augment the last figure , as you were informed in Multiplication. Example , let 34 be devided by 4'32648972101 , and let there be eight places of Decimall parts in the Quotient, having put to the worke a convenient company of Cyphers, it will be thus,

4'3264

4'32648972101) 34'00000000
20 28542804
371457196
346119178
25338018
21632448
3705570 (7'85856484
3461192
244378
216324
28054
25959
2995
1731
364
346
18
17
1

At every operation the Multiplees of the Divisor are diminished of a figure at the right hand untill the remayner come to 1, which is 0'00000001.

The same may be done without the bones, if you observe the proceeding of the Multiplees of the Divisor, they diminishing every time one place towards the left hand, which will be easie to doe, if the smaller Multiplees of the whole Division (when neede requires) be made equall in number of figures

gures to the greater, by prefixing Cyphers before them.

14. For distinguishing the Decimalls in the Quotient, observe this rule, that there be as many places of decimalls in the Divisor, and Quotient, as there are in the Dividend; placing Cyphers if neede be in the Dividend; yet the giving the Quotient his true denomination, when the Division is by Decimalls is sometimes troublesome and difficult; for often it happeneth that one or more Cyphers must come before the significant figures of the Quotient, which that you may easily doe, observe this rule. Marke unto what figure of the Dividend the last figure of the Divisor will come to, or fall under, then set downe the Index of that figure of the Dividend got by Sect. 31. chap. I. as also under it the Index of the last figure of your Divisor, then according to that which was before taught Sect. 1. chap. IIII. substract the Index of the Divisor from the Index of the Dividend, the remayning Index will shew of what quallity the first figure in the Quotient must be, as if the remayner be 2, then you must have 3 Integers before Decimalls, if a Cypher but one, if (1) then the first figure is a Decimall *prime* if (3) then two Cyphers must come in the Quotient next after the seperating line before any figure come in &c. And note that the operation is the same, what figures soever you take, if they stand the one under the other in the worke, as the Examples will shew you.

Examples

Examples of Division by Decimalls.

```
         (2)     (1)
28‘64)   774‘43692 (27‘040
         5728
         20163          (1)
         20048          (2)
          011569          1
          011456
            01132 &c.
```

Heere the Index of that figure of the Dividend to which the last figure of my Divisor falls under is (1) of the last of the Divisor is (2) which being set and substracted, the remaine is 1, which shewes there must be two Integers before any Decimalls come in.

```
                (4)      (1)                  (1)
Againe ‘2864)   774‘43692   (2704‘0    for    (4)
                                               3

                (1)      (3)                  (3)
Againe 286‘4)   7‘7443692   (‘027040   for    (1)
                                              (2)

                (0)      (4)                  (4)
Againe 2864)    ‘77443692   (‘0002704  for     0
                                              (4)
```

Another

Another Example.

$$\overset{(4)}{{}^{4}0235)}\quad \overset{(2)}{7{}^{4}82017}\ (345 \qquad \text{for } \frac{(2)}{\underline{(4)}}\ \ 2$$

This with a little practice will ſhew ſufficiently how to worke it; now I will ſhew you the practice in *Logiſtica Sexagenaria.*

Diviſion of *Sexagena's and Sexageſima's.*

15. Subſtract the Index of the Diviſor out of the Index of the Dividend, the remayne ſhews you of what kind the Quotient will be of, which if it be 2 it is *Sexagena's ſeconds*, if a Cypher degrees, if (1) *Sexageſima's primes &c.* *Example*, devide $45'''$ by $15'$.

Thus $15)\ 45'''\ (3''''$

(3) likewiſe devide $\underline{30^{\circ}.\ 36'.\ 48''.}$ by $6'$

$\frac{1}{(4)}$

$$\frac{6'}{5'.6''.8'''}$$

As by trying the Indices you may finde. The ſame worke doth ſerve to the Logarithemes, and to the powers of rootes &c.

The Proofes of Multiplication, and Diviſion are mutually by each other, for the Product divided by the

the one *Factor* brings the other *Factor* in the Quotient; so likewise if you multiply the Divisor, and the Quotient together (adding the remainder of the Dividend if any such were) the Product will be the Dividend.

17 And here (but that I have beene too long of this Subject) I might shew you how to divide by many numbers of two places all at once, as to divide any number by 25, is to cut off two of the last places (remembring to carry as many Unites to the next place, as 25 is contained in these two Figures, which can never be above 3, 9,) and redouble all the rest of the Figures for the Quotient, and from hence also you may divide any number by 24 (which will be of good use 240 d. being in a pound) all at once, if you first divide by 25, then see how many times 24 may be had in the Quotient, which in small numbers is soone perceived, which must be added to the former Quotient, the like might be done of many others, which for brevity sake (desiring to be briefe yet plaine) I leave to the practice of the ingenious Arithmetician.

18. The generall uses of Division are infinite, as to know the side of any *superficies*, the one side and content being given; to know the rate, price, or value of any thing, if the rate, price, or value of many of those things in the same kinde be given, it is of great use in the forming and working of any other rates, as of proportion &c. And if the diligent Reader be

but perfect in Multiplication and Division ; he may say he hath passed the hardest , the rest being all wrought by these 4 *Species* of *Numeration* , *viz.* *Addition*, *Substraction*, *Multiplication*, and *Division*

19. Lastly , note that if the Quotient be equall to the Divisor , and that nothing remayne , then is the Dividend a square number , and the Divisor the Roote. And note that if the Dividend be lesser then the Divisor , you set it in manner of a Fraction with a Lyne , otherwise you may convert it into a Decimall Fraction by putting Cyphers to it, and then dividing it.

And thus much of Division , now wee will proceede to Reduction, and some other usefull Rules for preparing the Reader to the Rules of practice.

CHAP. VI.

Of Reduction.

1. THough Reduction cannot properly be called any reall part of Arithmetick ; yet in respect for the avoyding of Fractions, they are either reduced into their least Denomination , or turned into Decimalls , whose Denominators are omitted,

I

I have thought good to place Reduction heere, which may be divided into two parts,

1. *Reduction of Fractions undenominate into their least tearmes, & contra.*

2. *Reduction of Fractions into Decimall, & contra.*

2. Reduction of Fractions undenominate, teacheth the manner, how to reduce or bring any summe, whether it be of number, weight, measure, or value from one denomination to another, retayning still the first value.

3. To reduce any Number or Fraction undenominate into his least tearmes, you must consider how many of the next lesser Denomination is contained in the next greater before, and by that Number multiply the greater, and so worke till you come to the least.

Example, to reduce 178 l. into Pence, according to the rule, I consider 20 s. is contained in one pound; therefore I double 178 l. it makes 3560, then because 12 d. is contained in one shilling, I multiply 3560 by 12, that is I double it, and adde it one place to the right hand, it gives 42720, which are the pence in 178 l.

If there be divers Fractions expressed of the same kinde, you must adde up the smaller Fractions, as followeth, 27 l. – 19 s. – 5 d. makes 6713 pence.

l.	s.	d.
27 —	19 —	5
20	540	6713
	559	
	12	

l.	s.	d.	ob.
3455	— 11	— 9 —	2
20	69100	829332	2317364
	69111	829341	2317366 p
	12	4	

Troy weight may be reduced into graynes, by multiplying by 12. 20. and 24.

Example, 352 l. makes 3548160 g.

4224
84480
24
168960
337920
3548160

Likewise 37—10.—16.—11 makes 218315 graynes.

l	oz	p	gr
37 —	10 —	16 —	11
12	444	9080	181920
	454	9096	36384
	20	24	218315
			gr

Averdepois great weight is brought into ounces by multiplying by 4. 28. and 16. As 7 C is 12544 ounces.

		C	q	l	oz.
7	In.	25 ——	2 ——	25 ——	10
4		4	100	2856	48096
28			102	2881	48106
28			28	16	ounces.
56			204		
224			816		
784					
16					
4704					
784					
12544 ounces.					

And note I multiply by 12 and 16 at one operation. I multiply by 24; First doubling; and double that againe, by 28, first double it for 2, then redouble it for 8. which practice will be found very easie.

The lesser Averdepois weight by 16 & 16. as 376 l. is 9600 drams

	l	oz:	dr.
So	244 ——	14 ——	11
	16.	3904	62688
		3918	62699 dr.
		16	

376
6000
9600

So of any other weight or measure, as to reduce miles into Barley Cornes. The distance betwixt *Yorke* and *London*, being 150, in Barley Cornes it is 28512000.

 First,

First, I multiply by 1760 yards in a mile, then these by 36 inches in a yard, then by 3 Cornes in an inch.

15|0	264|000	9504000
176|0	36|	3
15	792	28512000 bar. Corn.
90	1584	
105	9504000 inches	
264000 yards		

In time how many minutes in 1647 *Ægyptian* yeeres, I multiply by 365, then by 24, then by 60.

1647	711175	170682|00
365	24	6|0
8235	1422350	1024092 000 minutes.
5941	2844700	
11882	17068200 houres	
711175 dayes		

4 But there may yet be a shorter way to reduce the greatest into the least Denomination; by multiplying the greatest by as many as it contaynes of the least: For example, because in one pound there are 240 pence, therefore if you multiply pounds by 240 (which is nothing but to double twice) and put to the end a Cypher) they become pence.

In

In 375 l. there are 9000 pence in 35781. 858744 p.

24	24
750	71562
1500	143124
9000	

So multiply Pounds Troy by 1440 brings it into graynes; and so of the rest, as myles by 190080 Barley Cornes, &c.

5. Hitherto hath beene spoken of reducing the greater Denomination into the least; now to bring the least into the greatest and meanes, is by dividing the least, by so many thereof as the next greater Denomination contayneth: As to bring 42720 pence, I divide them first by 12, then by 20 thus

4) 42720
3) 10680 (3560 s. which divide by 20, a halfed
3) 3560 is 178 l. in 42720 d.

If any thing remayne in the first Division it is pence, in the second it is shillings. As what l. s. d. in 6713, answere 27 l. — 13 s. — 5 d. Or more speedily Pence may be reduced into Pounds by dividing by 240, the remayneder being shillings and pence.

After the same manner may Troy weight be reduced by dividing the graynes by 24, the penny-weight by 20, and the ounces by 12.

And so may Averdepois great or little, by dissolving them as you saw them made by Multiplication.

Likewise forraine Coyne may be reduced into *English*, first turning the value in *English Coyne* of any Part: As what is the value of 223 Scots Marks, because a Scots Marke is 13. ob. I multiply 223 by 54 the halfe pence in a Marke, the answer is 12042 farthings, which by Reduction dissolving is 12 l. — 10 s. — 10 d. — ob.

6. Hitherto hath beene spoken of Reduction of Fractions undenominate into their least tearmes & *contra*: Now come wee to shew how any Fraction, whether denominate or undenominate may be expressed in Decimalls; as for to reduce any Fraction into Decimalls, is to adde Cyphers to the Numerator, and to divide them by the Denominator, for if $\frac{3}{4}$ be to put into Decimalls: 4. 3 : : 1'0000 &c. Or as the Denominator is to the Numerator : : so any other Denominator is to a Numerator, which shall have the same value to the Denominator given as the first Numerator to his Denominator: And from hence you may know what part of one Fraction another may signifie, as what part of pence is the $\frac{3}{4}$ of a shilling. An 9 d, for as 4, 3 : : 12. 9 by multiplying 3 × 12, and dividing by 4. so $\frac{123}{782}$ in Decimalls is '1558 &c.

After the same manner 19 s. or $\frac{19}{20}$ is in Decimalls '95, of 16 s. it is 80, of 10 l. 5, of 5 s. '25, of 1 s. '05.

But if you be to put Fractions of Fractions into Decimalls; you must first reduce them into a single Fraction, which is taught in the Doctrine of Fractions; As for Fractions which wee call heere undenominate, being the famous parts of a pound weight or measure &c. reduce the Numerators into the least tearme of that compound Fraction given, and place for the Denominator the severall parts of the lowest Fraction contained in the Integer of the whole, as in one pound are contayned 240 pence, and 960 farthings, therefore if 2 s. and 5 d. were to be expressed in a Fraction it would be $\frac{29}{240}$ in Decimalls '1208 &c. Also 15 s. — 10 — ob. in a Fraction is $\frac{762}{960}$ in Decimalls it is '7937.

The like in Troy weight the Denominator of ounces is 12, of penny weight is 240, of graynes 1444, which may be brought into Decimalls by Reduction into the least tearmes, and then dividing them by the Denominator answering; the like of other weights and measures, and therefore $\frac{1}{4}$ of a yard, ell, &c. in Decimalls is '25. $\frac{1}{2}$. is '5. $\frac{3}{4}$ is '75

7. But because it will be somewhat tedious to convert Fractions into Decimalls this way; therefore it is best to prepare Tables of all these undenominate Fractions, out of which the Decimall Fractions may be gathered, as in the Tables following.

1. Table

1. Table.

Eng. Coyne.

s.	
19	·95000
18	·90
17	·85
16	·80
15	·75
14	·70
13	·65
12	·60
11	·55
10	·50
9	·45
8	·40
7	·35
6	·30
5	·25
4	·20
3	·15
2	·10
1	·05000

d.	
11	·04583
10	·04166
9	·0375
8	·03333
7	·02917
6	·025
5	·02063
4	·01667
3	·0125
2	·00833
1	·00417
f.	
3	·003125
2	·00208
1	·00104

2. Table.

Of English Coyne and Troy weight.

	f.	
	1	·0208
	2	·0416
	3	·0625
oz.	1 d	·083333
	1	·1041

2	· 1249	
3	· 1458	
2.d	· 16667	
1	· 1875	
2	· 2083	
3	· 2292	
3.d	· 2500	
1	· 2708	
2	2916	
3	· 3124	
4d	· 3333	33
1	· 3541	
2	· 3750	
3	· 3958	
5d	. 4166	67
1	· 4374	
2	· 4582	
3	5791	
6d	· 5000	00
1	· 5208	
2	· 5416	
3	· 5625	
7d	· 5833	
1	· 6041	
2	· 6249	
3	· 6458	
8d	· 6666	67
1	· 6874	
2	· 7082	
3	7291	
9d	· 7500	00
1	· 7708	
2	· 7916	
3	8225	
10d	· 8333	33
1	· 8541	
2	· 8749	
3	8958	
11d	. 9166	67
1	· 9374	
2	· 9582	
3	· 9791	

Penny weight.

p.		
19	· 0791	67
18	· 075	
17	· 0708	33
16	· 0666	67
15	· 0625	
14	· 0583	33
13	· 0541	33
12	· 0500	
11	· 0458	33
10	· 0416	66
9	· 0375	00
8	· 0333	33
7	· 0291	66

6	· 0250
5	· 020833
4	· 016667
3	· 012500
2	· 008333
1	· 004166

Graines.

gr.	
18	. 003125
12	· 002083
6	· 001041
5	· 000868
4	· 000694
3	· 000520
2	· 000347
1	· 000173

3. Table.

Averdepois great weight.

$\frac{3}{4}$ C:	·75000
$\frac{1}{2}$ C:	· 50000
$\frac{1}{4}$ C:	· 25000

l.	
27	· 24107
26	· 23214
25	· 22321
24	. 21428
23	. 20535
22	· 19642
21	· 18850
20	· 17857
19	· 16964
18	· 16071
17	· 15178
16	· 14285
15	. 13392
14	· 12500
13	· 11607
12	· 10714
11	· 09821
10	· 08928
9	· 08035
8	. 07143
7	· 07250
6	· 05357
5	· 04464
4	· 03571
3	· 02678
2	· 01785
1	: 00893

l.
3 q. · 00670
l.
2 q. · 00446
l.
1 q. · 00223

Averdepois little weight.

oz.
15 · 9375
14 · 8750
13 · 8125
12 · 7500
10 · 6250
9 · 5625
8 · 5000
7 · 4375
6 · 3750
5 · 3125
4 · 2500
3 · 1875
2 · 1250
1 · 0625
3 q · 046875
2 q · 03125
1 q · 015625

4. Table.

Of Time.

11 · 91667
10 · 83333
9 · 75000
8 · 66667
7 · 58333
6 · 50000
5 · 41667
4 · 33333
3 · 25000
2 · 16667
1 · 08333

Dayes.

30 · 08219
25 · 06849

20	·05480
15	·04109
10	·02734
5	·01369
4	·01096
3	·00822
2	·00548
1	·00273

3q	·00204
2q	·00136
1q	·00068

5. Table.

Of measure, viz. of Yards, Ells, &c.

3q	·7500
2q	·5000
1q	·2500
3.	·1875
2.	·1250
1.	·0625

To make the Decimall Tables abovesaid, for shillings, pence, and farthings.

First, conceive 1 l. or 20 s. to be divided into 10000, and then, saying, if 20 be equall to 10000, then 10 s. the ½ of 20; is equall to ·5 the halfe of 10. and so 5 s. the ½ of 10 s. equall to ·25 the half of ·5 or ·50. and ,05 equalls 1 s. the fift of 25 is 5 s. adde up the intermediate spaces

s	
20 =	1·00000
10 =	·5
5 =	·25
1 =	·05

of shillings, you may make the Table for shillings, which if you conceive a Cypher placed at the Number of any summe of shillings, and take halfe there-of

of you have the Decimall anſwering.

Now to make the Table of pence, you ſay if '05 be equal to 12 d. then '025 equal to 6 and 1004166 &c.

d	
12 =	'05
6 =	'025
3 =	0215
1 =	004166 &c.

equall one penny, which is no pure Decimall, but may be continued at pleaſure, Now the intermediate places of pence are made up by Addition or Subſtraction, thus take the Decimall anſwering 1 d., and ſubſtract it from that anſwering 3 d. it leaves the Decimall of 2 d. to be '008234. ſo of 4 d. to be 016666, and ſo of the reſt, alwayes noting thus much that a perfect Decimall (which is alwayes one that ends with 5) may be one of the two you adde or ſubſtract together.

f	
4 =	'0041666
2 =	'00208333
1 =	'00104167

Againe if 4 d. or 4 f. be equall to '004166, then 2 f. equall to ,002833, and 1 f. = '00416, and the 3 d. farthing not expreſſed is equall to '003125 by adding up 2 and 1 farthings, and after this manner is the firſt Table of Decimalls in coyne, made, where note this, that as all perfect Decimalls end with 5, and expreſſe the true value of the Fraction; ſo all other Decimalls are imperfect, not expreſſing the true value, though they want little of truth, to the [illegible] whereof you may continue them to what places you pleaſe, as 1 d. = 00416, ſo likewiſe 1 d. = 004166666, and alwayes in the end of every Decimall

mall if the figure following it, be either 5 or above, augment the last figure by 1, so if you will use 6 places 1 d. = 004167 the which you will observe in all the Tables following.

7. But because pounds are so easily reduced into shillings by doubling, and shillings into pounds by halfeing; therefore it is far better to make one shilling the Integer, and from that the second Decimall Table of English Coyne is made, with the farthings put betwixt each penny; so that this way the pence and farthings are suddainely expressed in 3 or 4 figures: The making this Table is after the same manner that the last was, conceiving 1 s. or 12 = 1000.

s.	
12 =	1'000
6 =	'500
3 =	'250
1 =	0833
2f =	0208
1f =	0208

And the intermediate places with farthings are made up by Addition and Substraction as before.

And the Decimalls in the second Table signifying Pence, and printed in a greater Character may likewise signifie the Ounces of one pound Troy the pound being 12 ounces answering twelve pence of a shilling, the penny weight of the pound Troy are in the third Table, and each sixt Decimall of graynes.

The third Table contaynes the Decimalls of Averdepois great and little weight; in the first Hundreds are Integers, and quarters, pounds, and quarters are in Decimalls, in the second, ounces and quarters in Decimalls.

Hara The

The fourth Table consists of the Decimalls of Time, wherein a yeere is the Integer, the moneths, and each fifth day the Decimall.

The fifth the Decimall of a yard or ell: The making of these Tables are as that of shillings, therefore for your own use you may make Tables for all other weights and measures, these will serve for our purpose.

9. The use of these Tables are to expresse suddainly the Decimall, answering to any of the undenominate Fractions; for if you take the Decimall answering the Fraction propounded, or if the Fraction be compounded, then adde the Decimalls together, it shall expresse the value of that Fraction. As the value of 15 s is '75. of 9 d. is '0375. of 10 oz. troy is '83333 of 7 s. is '35 of 2 f. is '00208. of 5 moneths '4166 of $\frac{1}{4}$ ell is '25 of 10 dayes '0273

Also of 15 s. 9 d. ob: is '78958 of 9 oz. 16 p. is '8166

15	'75		9	'75
9	'0375		16	'0666
2	'00208			
	'78958			

But for ready practice, I advise the Reader to use the second Table of Coyne; where you may have the

 penc

pence and farthings at one worke, only remembring to double the pounds if any such be with a Cypher, and adde them to the shillings: As for Example.

15 s. — 5 d. in Decimalls is 15'416. and 31 l. 12 s. 11 d. is 632'916; so 1521 l. — 11 s. 5 d. — 2 f. in Decimalls is 3051'458, as appeares by the Table.

10. Thus have I shewed how to reduce Fractions whether denominate or undenominate into Decimalls; now I shall shew how to know the Fraction or value expressed by a Decimall, the rule thus: multiply the Decimall by the Denomination of the Fraction desired, and that which exceedes the *Seperatrix* is the Numerator of the new Denominator: for example I would know the Numerator to the Denominator 4, if the Decimall be '75, I multiply 75 by 4, make 3'00; therefore it signifieth the Fraction $\frac{3}{4}$ for as 1'0000. '75 :: 4. 3. so '1558 would be $\frac{123}{789}$.

11. And if you be to reduce a Decimall into another Fraction, whereof there be divers Denominations, you had best doe it by the Denominator of the least parts, according as was premised in Sect. 5.

12. But if you either make Tables as I have directed Sect. 7, or make use of the Tables made, to expresse the value of any Decimall Fraction, you must doe thus:

If the Question be in the first Table of shillings, pence,

pence, or farthings; first seeke out what shillings are intimated by the two first figures of the Decimalls, and setting downe the shillings expressed, then substract the Decimall expressed by shillings out of the Decimall given , and with the Remayner enter the pence, &c; As for example,

‘35 I finde to signifie 7 s. in the first Table, and
‘0375. 9 d.

Also ‘78958 signifieth 15 s. — 9 d. — ob.
‘75 = 15 s.
‘03958 = 9 s. Likewise ‘55321 = 11 s. – 1 f.
‘0375 ‘55 = 11 s. – 1 f.
00208 = 2 f. ‘00221 = 1 f.

Also 32‘5785 = 32 l. – 11 s. — 6 d. — 3 f.
‘55 = 11 s.
‘0285 = 6 d.
‘025
‘0035 = 3 f.

But if you use the second Table, then you are to account the Integers shillings, reducing them to pounds by halfeing the summe, and taking out the pence and farthings answering the Decimalls.

So 3257‘982 signifieth 162 — 17 s. — 11 — 3 f.
So 13250‘416 signifieth 662 l. — 10 s. — 5 d.

Likewise 3051'458 signifieth 152 l.–11 s.–5 d. ob.

If the Question be of Troy weight, then worke as before. First with Ounces, then with Penny-weight, and graines: As 32'8166 signifieth 32 l.–9 oz.–16 p.

'75 = 9 oz.

'0666 = 16 p.

The same may be said of Time, or of any other Fraction you make Tables for.

13. And although it be heere a little from the matter, yet I will heere shew you what the aliquot parts of a pound or shilling are, which are of good use in the Rules of practice.

The aliquot, or even part of any thing, is such a Fraction or part of it, that having 1 to his Numerator may have a Denominator, having the same value with the former Fraction: For as the Numerator is to the Denominator, so 1 is to the new Denominator.

Therefore the Denominators may expresse the aliquot parts, as 2 or $\frac{1}{2}$ may expresse 10 s. for as 10. 20 : : 1. 2. or 6 d. for as 6. 12 : : 1. 2. And therefore in this Table are expressed the aliquot parts both of a pound and shilling.

The

The Aliquot parts of a pound

s	d	Parts
0	0	
1	3	16
1	4	15
1	8	12
2	0	10
2	6	8
3	4	6
4	0	5
5	0	4
6	8	3
10	0	2
20	0	1

The Aliquot parts of a shilling are

d	Parts
6	2
4	3
3	4
2	6

And thus much of Reduction, now come wee to Proportion.

CHAP. VIII.

Of Arithmeticall, and Geometricall Proportion continued.

1. HItherto hath beene spoken of the simple parts of Arithmetick; now come wee to the Comparative part, which wee only divide into Arithmeticall, and Geometricall Proportion, and first;

2. *Arithmeticall continued Proportion*, is a continued *Progression* or *Series* of Numbers, increasing or diminishing by equall differences, as the Numbers 3. 5. 7. 9. 11 &c. are continued in an Arithmeticall Proportion, the common difference being 2.

3. Heere wee are to consider five things. (1) The first and least tearme 3. (2) the last and greatest teame 11. (3) The number of tearmes which are heere 5. (4) The common difference 2. (5) The summe of all the tearmes 35. And note that the number of the differences is the number of the tearmes made lesse by one, and that the summe of the differences is equall to the Product of the Number of the tearmes drawne into the common difference, made lesse by that common difference which is equall to the last tearme, made lesse by the first.

The Considerations.

(1) *The first tearme.* (2) *The last tearme.*

(3) *The number of tearmes.* (4) *The common difference*

(5) *The summe of all the Tearmes,*

(4) Having three of any of these five given, the other two are found out by these twenty Rules following; invented by Mr. *Oughtred* in his *Clavis*, Chap. XVIII. Prob. VI. the which in words are these.

(1) *Having*

(1) *Having the 1. 2. and 3 given to finde the 5.*

To the Product of the 3 into the 2, adde the Product of the 3 into the 1, the summe of these 2 Products divided by 2, is equall to the 5.

(2) *Having the 1. 2. and 3 given, to finde the 4.*

Divide the 2 d. made lesse by the 1 st. by the 3 d. made lesse by unity or one, and the Quotient is equall to the 4.

(3) *Having the 1. 2. and 4 given to finde the 3.*

Substract the 1 st. from the second, divide the remainer by the fourth, and adde unto the Quotient unity, the summe is equall to the 3 d.

(4) *Having the 1 2. and 5 given, to finde the 3.*

Double the 5, and divide the summe by the 1 added to the 2, the Quotient is equall to the 3.

5. *Having the first, 3 d. and fourth given to finde out the 2 d.*

If you multiply the 3 d. and fourth, and take from the Product the fourth, to the Remaines adde the first, the summe is equall to the 2 d.

6. *Having the first 3 d. and fourth given, to finde out the fifth.*

Multiply the 3 d. and fourth, from the Product take the fourth, to the remaine adde the double of the first, the summe multiplyed by the 3 d. and divided by two, is equall to the fifth.

7. *Having the first 3 d. and fifth given, to finde out the 2 d.*

Double the fifth, and substract from it, the Pro-

duct of the 3 d multiplyed by the first, divide the remaines by the 3 d. the Quotient is equall to the 2 d.

8. *Having the 2 d. 3 d. and fourth given, to finde out the first.*

From the sum of the 2 d. added to the fourth, substract the Product of the 3 d. multiplyed by the fourth, the remaynder is equall to the first.

9. *Having the 2 d. 3 d. and fourth given, to finde out the fifth.*

From the doubled summe of the 2 d. added to the fourth, substract the Product of the 3 d. multiplyed by the fourth, and multiply the Remaynder by the 3 d. the Product is equall to the fifth.

10. *Having the 2 d. 3 d. and fifth given, to finde out the first.*

Double the fifth, divide it by the 3 d. substract from the Quotient the 2 d. the Remaynder is equall to the first.

11. *Having the 2 d. 3 d. and fifth given, to finde out the fourth.*

Double the Product of the 3 d. multiplyed by the 2 d. and substract from it the double of the fifth, divide the Remayner by the 3 d. multiplyed in it selfe, and made lesse by it selfe, the Quotient is equall to the fourth.

12. *Having the first 2 d. and 4th. given, to finde out the fifth.*

Substract the square of the first, from the square of

of the 2 d. divide the Remayner by the fourth, to the Quotient adde the first and 2 d. the halfe of the sum is equall to the fifth.

13. *Having the first 3 d. and fifth given, to finde out the fourth.*

Substract the square of the first from the square of the 2 d. divide the Remayner by the fifth doubled, made lesse by the first and 2 d. the Quotient is equall to the fourth.

14. *Having the first 3 d. and fifth given, to finde out the fourth.*

Double the first, and substract it from the fifth doubled, divide the Remayner by the square of the 3 d. made lesse by it selfe, or by the 3 d. the Quotient is equall to the fourth.

15. *Having the 3 d. fourth and fifth given, to finde out the first.*

Divide the fifth doubled, by the double of the 3 d. adde to the Quotient halfe of the fourth, and substract from that summe the halfe of the product of the 3 d. multiplyed into the fourth, the Remayner is equall to the first.

16. *Having the 3 d. fourth, and fifth given, to finde out the 2 d.*

Divide the fifth doubled, by the double of the 3 d. adde to the Quotient the halfe of the Product of the 3 d. multiplyed by the fourth, and from that summe substract the halfe of the fourth, the Remayner is equall to the 2 d.

The

The former 16 Prop. may be all wrought by the rules before going, the latter foure are wrought by extracting the ſquare roote taught in the ſecond part of this booke.

17. *Having the firſt, fourth and fifth given, to finde out the 2 d.*

Take the fourth part of the fourth, double the Product of the fifth multiplyed by the fourth, and ſquare the firſt (or multiply it into it ſelfe) adde theſe three numbers together, and ſubſtract from the ſumme the 4: if you take the ſquare roote thereof and ſubſtract halfe of the fourth from it, the Remayner is equall to the 2 d.

18 *Having the firſt, fourth and fifth given, to finde out the 3 d.*

Double the firſt, and ſubſtract from it the fourth, keepe the Remayner in minde. Then,

Square that remayner in minde, divide it by the ſquare of the fourth redoubled, from the Quotient ſubſtract the Quotient of the fifth doubled divided by the fourth; now if you take the ſquare roote of the laſt Remayner, and ſubſtract from it the Quotient of the firſt Remayner in minde, divided by the 4 doubled, this laſt Remayner is equall to the 3 d.

19. *Having the 2 d. fourth and fifth, to finde out the firſt.*

If you adde together the fourth part of the ſquare

square of the fourth, and the square of the second, and the Product of the second and fourth multiplyed together, and from this summe if you substract the Product of the fourth and fifth doubled, and lastly if to the square roote of the last Remayner, you adde the halfe of the fourth, the summe is equall to the first.

20. *Having the second, fourth, and fifth, to finde out the third.*

If you divide the double of the second added to the fourth by the fourth doubled, and take from this Quotient the square roote of the number following, (*viz.* the Quotient of the redoubled square of the second, added to the Product of the fourth multiplyed, by the second redoubled, added to the square of the fourth, made lesse by the Product of the fifth doubled, multiplyed by the fourth redoubled, and divided by the square of the fourth redoubled) the Remayner is equall to the third.

I have expressed these in as short a forme as well I could, the excellency of Mr. *Oughtreds* way, and his Invention may plainely appeare by them, they being all contained in the *Clavis* in 2. 8° Pages. Therefore,

3. When any question resolvable by Arithmeticall Progression is propounded, consider on which of the five president considerations the resolution depends, *viz.* whether the demand be to give the first or last tearmes the number, generall difference, or

or summe of the tearmes, that is whether of the first, second, third, fourth or fifth.

If it be of the first, then the rest of the tearmes given, and the resolution of the Question will depend either on the eight, tenth, fifteenth, or nineteenth precedent Rules.

If of the second, it depends on the fifth, seaventh, sixteenth or seaventeenth.

If of the third, it depends on the third, fourth, eighth, or twentieth.

If of the fourth, it depends on the second, eleaventh, thirteenth, or fourteenth.

If of the fifth, it depends on the first, sixth, ninth, or twelfth.

The resolution of the worke when you have duely stated the considerations, is found out by the Rule.

Example 1. If the first tearme of a Progression be 5, the last tearme 15, and the number of the tearmes 6. what is the summe of the tearmes, and what the equall difference of each tearme: Heere is given the first, second, and third considerations to finde the fifth, and fourth, which by the first and second Rules give the summe to be 60. and the difference 2.

The worke.

6× 6× 90 | 120 (60 *the sum.* 15-6 | 10 (2 *the difference.*
15 5 30 | 2) 5 1 (5
90 30 120 | 10 5 |

Example 2. Of eight Brethren, the yongest was 27 yeare s old, the eldest 50, each differed like in age from other, what difference was there in their ages, and what the age of each brother? (3) The number of tearmes 8. (1) the first tearme 27. (2) the last tearme 50 are given to finde out (4) the common difference by the second Rule the worke finds the equall difference to be $3\frac{2}{7}$.

$$8-1=7)\ 50-27=23\ (3\tfrac{2}{7}-$$
$$-\textit{for}\ 3\tfrac{2}{7}\times 7=23\ \&\ 23+27=50$$

Example 3. One had divers Sonnes, the yongest 6 yeares old, the eldest 40, and every one in order still exceedes his brother by 2 yeeres; How many Sons had he? *An* 18. And how much was the number of all their Ages? *An.* 414.

Heere is the first, second, and fourth given to finde the third, and fifth.

By rule 3 d. 2) 40 - 6 = 36 (17 + 1 = 18.

By rule 12th. 1600
36
2) 1564 (782
6
40
2) 828 (414. (5)

Catherine Palmer her

Example

Example 4. One travelled 50 miles every day, increasing his journey 2 miles, till at 5 dayes end he finished his journey ; how many miles was his first dayes journey ? Answer 6. And how many the last? Answer 14.

In this question the third, fourth, and fifth are given to finde the first by rule 15. and the 2 d. by rule 16.

$$10)\ 50+50 = 100\ (10+1 = 11$$
$$5\times2 = 10\ (5 = \frac{5}{6}\ (1)$$
$$2)$$

$$10)\ 50+50 = 100\ (10$$
$$5\times2 = \frac{10}{2} = \overset{5}{—}\ 15-1 = 14.\ (2)$$

Example 5. One had 20 clothes worth 5 l. a peece ready money, and sold the first for 1 crowne, and augmented his price 2 crownes more for every cloth then was payd for the former, what received he for his cloathes, and whether did he gaine or lose of the just price of 5 l. a cloth? *Answ.* he received 300 crownes or 750 l. his cloathes at 5 l. the cloth, comming to 100 l.

Heere the third (20) is given, the first (1) and the fourth (2) by the sixth Rule.

$$40+2-\underline{2} = \underline{40}\times\underline{20} = \frac{600}{2} = 300.$$

Example

Example 6. One hundred eggs are placed every one yard distant from another, and 1 yard distant from a basket; whether might one gather up the eggs one after another, still returning to the basket and putting them in, before another doth run 4 miles or 6040 yards? Answere he that gathered the eggs went 10000 yards, which exceedes 4 miles by 3960 yards.

Heere the first, third, and fourth tearmes are given to finde out the fifth, by the sixth Rule.

$$100 \times 2 = 200 - 2 + 2 = 200 \times 100 = \frac{20000}{2} = 10000.$$

Example 7. A summe of money is to be payed in 12 dayes, paying the first day 10 l. and increasing e-very payment after 5 l. till the dayes be expired, what is the whole debt? Answere 168 l.

$$12 - 4 = 8 + 20 = 28$$

$$28 \times 12 = \frac{336}{2} \ (168.$$

$$\begin{array}{r} 12 \\ \hline 336 \end{array}$$

Example 8. Six hundreth eighty five pounds is disbursed by Arithmeticall Progression, to how ma-ny I know not, but the first have 19 l. and the last at 18 l. How many did receive the money? And how much had each? Answere, There were 10 to receive the money.

The

The common difference being 11 l., each mans summe may be easily had by adding 11 to the first mans who had 19 &c.

$$685 \times 2 = 1370\,(10$$
$$118 \times 19 = 137)$$

118	19	13924	685	
118	19	361	2	
944	171	13563	1370	
1298	19		127	
13924	361		1233)	13563 (11
				1233
				1233
				1233

After this manner may such questions of Arithmeticall continued Proportion be wrought.

Geometricall Proportion continued, is when any *series* of Numbers set downe, have the same or equall reason, that is the Quotients of each following tearme divided by the former are equall; as. 2. 4. 8. 16. &c. the *Ratio* being 2 for 16 containes 8 twice, and so 8, 4 and 4. 2.

6. In Geometricall Proportionalls continued, the first and last are commonly called the extreames, and all the middle tearmes are called the meanes.

7. Because the finding the meanes in a Progression Geometricall is done by extracting the Rootes, not

Thō Spurling John

yet taught, and because the great use of this proportion is in satisfying questions of compound Interest, which are all resolved by tables made for that purpose, explaned in the end of this booke, I referre you to the same; onely take this rule for the finding the summe of any progression Geometricall.

Multiply the last tearme by the same *Ratio* found by Sect. 5 from the Product take the first tearme, divide the remaynder by a number lesse by 1 then the *Ratio*, the Quotient is the summe of that Progression.

1. Ex. 3. 6, 12. 24. 48. the summe is 93 for the *Ratio* being 2. 48×2 = 96 − 3 = 93 divided by 1 equall 93.

Therefore for resolving of such Questions where nothing but the first tearme *Ratio* and number is given, you must first finde the last, and then the summe as before, the last tearme or any other tearme may be discovered by placing Arithmeticall tearmes over the heads of the Proportion, and continuing some few of the first of the Progression.

Ex.	1.	2.	3.	4.	5.	6.	7.	8	*Arith. Pro.*
	2.	4.	8.	16.	32.	64.	128.	256	*Geo. Pro.*

The lower numbers 2. 4. &c. are in Geometricall proportion, the higher in Arithmeticall proportion; and note that for the desired tearme if it be greater then any given, adde up such two of the Arith. Prop.

as will make up the desired tearme, and multiply the Geo. tearmes under the Arith. it will satisfie your desire.

As suppose in the former progression I have only 4 of the first tearmes given, and the 10 tearme is desired:

I finde $2+4+4=10$ *The* 10 *Tearme in Arith.*
$4\times16\times16=1024$ *The* 10 *Tearme in Geo.*

But if the Tearme desired be lesse then a tearme given, substract the Arith. tearmes from the given, and divide the Geo. answering you have your desire. As in the former progression the 3 first tearmes $\begin{smallmatrix}1. & 2. & 3.\\ 2. & 4. & 8.\end{smallmatrix}$ and the eight tearme $\begin{smallmatrix}8\\ 256\end{smallmatrix}$ were given to finde the fifth tearme.

$8-3=5$
$8)\ 256\ (=32$
$\underline{24}$
16

The same manner of worke may be observed in the finding any tearme proposed, in what Question soever.

Ex. 1. One bought a Horse after this manner. The horse had 4 shooes, every shooe contayning six nayles in all 24 nayles, he thinking to have a good bargaine was content to pay a farthing for the first nayle, and double it to the last, what ought he to pay for the horse? Answere.

4+5

$$4+5 = 9+9 = 18+5 =$$
$$16 \times 32 = 512 \times 512 = 262144 \times 32 =$$
$$= 23+1 = 24$$
$= 8388608 = 24$ Or last tearme.

Then by the Rule Sect. 7. $8388608 \times 2 = 16777216 - 1 = 16777215$, which is the summe of the Progression, and the number of farthings he ought to pay for the horse, which make 17476l. 5s — 3d. 3f. a very great rate to give for a horse.

Chap IX.

Of Geo. Proportion discontinued, or of the Golden Rule.

1. IF of foure numbers the first be to the second, as the third is to the fourth, that is if the Quotient of the former two divided, be equal to the Quotient of the latter two divided, those foure Numbers are said to be proportionall.

2. Therefore if foure numbers be proportionall, the product of the meanes is equall to the Product of the extreames. As if 2. 4 : 8. 16. 2×16 is equall to 8×4 viz. 32.

3. Therefore if the Product of the two meanes be divided by the first, the Quotient is equall to the last number; or tearme, as $4 \times 8 = 32$ which being divided by 2 the Quotient is 16 the last tearme.

4. From hence ariseth the knowledge of a fourth number which shall have such proportion to one number given, as other two numbers given have the one to the other, and this is called the Rule of Three, or for the excellency thereof the golden rule, and herein we are to consider the Division, Disposition, Operation and Use.

5. The rule of three or golden rule is either simple or compounded, the simple is either direct, or indirect, and the compound rule is likewise direct indirect, ascending or discending.

6. The direct rule of three is, if the second tearm be greater then the first, the fourth tearme shall likewise be greater then the third, if lesser then lesser: Or in the question, if more require more, or lesse lesse, the question is to be answered by the golden rule direct, as if 4 yards cost 12s. then 8 yards must needs require more, or a greater number of shillings.

7. The manner of placing the tearmes for worke is after the first tearme to subscribe a point, after the second foure, after the 3 one as here. 1. 2 : : 3. 4.

And by this meanes the Rule may be expressed in one or two lines. In this rule that which you are mainely to consider is, the 1st. and 3d. tearme are alwayes of one Denomination or name, and s

ar

are the 2d. and fourth, as if the first be money weight measure time &c. of the same name is the 3, and so must the fourth, which is still sought, be of the second, and the tearmes of the rule are to be read or understood so accordingly.

Ex. 3l. Pepper. 18s : : 5. 30 to be read thus, if 3l. of Pepper cost, or will give 18s. then 5 pound of Pepper will give or cost 30s.

6. Omit in setting downe the question that which is common to both sides of the Figure, as if 20 tun of wine cost 4d. for the carriage of it 6 miles, then what will the carriage of 20 tun 12 miles cost? Here 20 tunne is to be omitted, and thus the question will stand. 6. 4 : : 12.

7. The three numbers given are so to be disposed, that the number whereof the value or price is sought, or the number demanding, is to be placed in the 3 place, or the first tearme in the second part of the Figure, the tearme that is of the same denomination which is set in the first place, and the number remaining set in the 2d. place], of which nature the number sought must ever be: As in this example, what will 9 yards cost, if 2 yards be bought for 5s. 2. 5 : : 9. 22'5.

If 12 yards of Damaske will lyne 16 yards of Velvet; how much Damaske will lyne 24 yards of Velvet? Thus to be set downe. 16. 12 : : 24. 18. And if the first and third tearmes be of unlike names they are to be reduced or put into Decimalls, accor-

 ding

ding to the Rules in the VII. Chapter.

8. After you have thus orderly disposed the places, for finding out the fourth tearme; *Multiply the second and third tearmes together, and divide the Product by the first tearme, the Quotient is the fourth tearme*, And

Therefore if an unite be in the first place, the fourth tearme is gotten by Multiplication.

Therefore if an unite be in the second or third places of the rule, the fourth is gotten only by Division.

Rules of practice performed by Multiplication, 1 being the first tearme in the Rule.

The Questions and Operations.

At 6d. the yard, what 575 yards give? Answere, 14l. 7s. 6d.

1 yard. 6d : : 575. 287$\frac{6}{5}$ or 14l. – 7s. 6d.

The Explanation.

This is wrought by taking halfe 575 . 6. being $\frac{1}{2}$ of a shilling.

At 9 the yard, what 5271 yards? Answer, 197l. — 13s. — 3d.

1. '75 :: 5271. 3953'25

```
      75
   26355
  36897
  3953'25
```

This is wrought by taking the Decimall of 9d. out of the second Table.

At 5d. ob. the ell, what 120 ells? Answer, 2l. — 15s.

1. '4582 :: 120. 54'984+

```
       4582
      91640
     4582
    54'9840
```

Answered by Decimalls it might have beene answered by the third part $\frac{1}{3}$, and eighth part in Aliquots.

At 16s. the pound, what's 2312l.? Answere, 1849l. 12s.

1. 16s :: 2312. 1849l. 12.

$$\begin{array}{r} 2312 \\ 16 \\ \hline 1849.\ 12. \end{array}$$

Heere the worke is brought into pounds and shillings all at once, by taking the halfe of the Multiplyer after the worke of the first Figure.

At 2s. 5d. 1 f. the yard, what is 123 yards at? Answere 14l. 19s. 9d. $\frac{1}{2}$

$$\begin{array}{lrl} 1. & 2'4375 & :: 123.\ 299'8125 \\ & 123 & \\ \hline & 24375 & \\ & 48750 & \\ & 73125 & \\ \hline & 299'8125 & \end{array}$$

This is wrought by Decimalls in the 2d. Table.

At 1d. the pound, what 235781l.? Answere, 982l. — 15s. — 1d.

$$\begin{array}{lrl} 1. & 1 :: & 235781. \\ & 4) & 58945'25 \\ & 3) & 19655'0833 \end{array}$$

This wrought by dividing the number by 4 and 3, to

to bring it into shillings, which is speedily done, and to be readily practised,

At 9d. the pound, what doth 1578 ¼ come to? Answere, 59l. — 3s. — 8d. ½ f.

1. 9 : : 1578'25. 1183'6875
 4) 14204'25
 3) 3551'0625
 1183'6875

At 7s. — 11d. the ell, what will 856 ells cost? Answere, 338l. — 16s. — 8d.

1. 7'9167 : : 856. 6776'6952
 856
 475002
 395835
 633336
 6776'6952

When you have multiplyed them, turne the Decimalls into pence by the second Table, and the shillings into pounds as above.

At 5s. — 1d. — ob. an ounce of Plate, what will 356 ounces cost? Answere 91l. — 4s. — 6d.

1. 5'125 :: 356. 1824'50

```
      756
  15375
   30750
  25625
  1824'500
```

After the Multiplycation, the pounds by halfeing, and the pence out of the Table are had at the first sight.

At 2s. — 5d. — 1f. the day, what will 365 dayes come to? Answere, 44l. — 2s. — 1d.

1. 2'4167 :: 365. 882'0955

```
       365
  120835
  72501
  145002
  882'0955
```

At 3l. — 15s. — 9d. the ounce of Gold Plate, what will 175 ounces come to? Answere. 662l. — 16s. — 3d.

1. 75'75 : : 175. 13256'25
175
37875
53025
07575
13256'25

What is 312 ¾ ells at 12s. — 6d. worth? Answere 195l. — 9s. — 4d. ob.

1. 12'5 : : 312'75. 3909'375
12'5
156375
062550
031375
3909'375

What is a wedge of gold worth that weigheth 4 ounces, 6p: 15gr: at 37l. 17s. — 10d. the pound? Answere, 13l. — 13s. — 5d. — 2f.

1. 757'803 : : '3608. 273'4261
3638
6062664
227349900
4546998
273'4261464

'3333 = 4oz.
'025 = 6p.
'0025 = 16gr.
'3608

Here

Heere observe that I put the Question into Decimalls, by taking out of the Table of Troy weight the Decimalls answering 4oz: 6p: 15gr: and then adding them together, I multiply or double the pounds to bring them into shillings, which are added to the shillings at one worke.

What is a Chest of sugar worth that weigheth 8 ½ C 13l. at 10l. 5s.-7d. the C? An.88l.— 11 – 4d.

1. 205'583 : : 8'6161. 1771'3237
 8'6161
 0205583
 1233498
 0205583
 1233498
 1644664
 1771'3237

½ C = '50
13l = '1161
 '6161

A parcell of Bullion is bought at the rate of 84l.-16s.-8d. the l. what is that an ounce? An.7l.-1s.& 4d.

1. 1696'667 : : '0833. 141'3323
 '0833
 05090001
 05090001
 13573336
 141'3323

Unto

Unto how much Sterling money doe 324 Duccats amount at 7s. 5d. sterling the peece? Answere, 120l. 5s. & 1d.

1. 7'416 :: 324. 2403'108

324

14834

29668

22251

2403'108

If a *Philips* Dollar be worth 4s.-11.-ob. what are 781 worth? Answere, 193l.-12s.-6 $\frac{1}{2}$

1. 4'96 :: 781. 3872'76

4'96

3124

4686

7029

3873'76

What is a graine of Gold worth at 37l.-17s.-10d. the pound? Answere, 1d. 2f. $\frac{1}{2}$.

1. 757'8333 :: '000173. '1311

'000173

22734999

53048331

07578333

'1311

In these questions where it is required to cut off some Number of decimalls, the way for Contraction of Multiplication before set downe is to be practised.

If a C of sugar cost 6l. – 13s. what shall 1l. cost? Answere, 1s. – 2d. f.

1. 133 : : '00893. 1'1876.
133
2679
2679
0893
1'18769

If one spend in a yeere 397l. - 18s. and 11d. what is that a day? Answere, 1l. 1s. 8d. 3f.

1. 7958'916 : : '00273. 21'187277
'00273
:0:795:8'916
2387:
55712:
159178:
217' 277:

[Ru]les of practice performed by Division. An Unite or 1 being in the 2 or 3 place of the Rule.

If a peece of 45 yards cost 3l. – 16s. what is that yard? Answere, 1s. – 5d. ob.

45. 76 :: 1. 1'4666

0	45)	76 (1'4666
0	90)	45
•	180	21'0
	360	180
		3'00
		270
		300

If 123 $\frac{1}{4}$ yards of Velvet cost 91l. – 18s. – 6d. what will one yard cost? Answere, 14s. – 10d.

123'25). 1838'5 :: 1. (14'834

	12325
(1)	606'00
(2)	49300
1	103'000
	98700
	043000
	36975
	60250

If I buy 31. 5 ounces of gold Plate for 136l. 15s. 4d. what may I sell one pound for after that rate? Answere 40l.

31'4166.) 2733'3333 : : 1. (800'00
2733 28
000005

If an ounce of Silver be worth 5s. - 6d. what is a pound of the same worth? Answere 3l. — 6s. — 1d.

'0833). 5'5 : : 1. (66'024
4998
(1) 5020
(2) 4998
1 02200
01666
03340

First wee put the ounce Troy into Decimalls, and likewise 5s. - 6d. Then in Division for finding the denomination of the Quotient; I substract the Index of 8, which is (2) from 5 above it which is (1) the remaine is 1, which shewes 2 Integers in the Quotient.

If 37 ounces 17 p. and 10 graines of white plate cost 13l. 13s. 6d. 1f. what will a pound of that plate cost? Answer, 5l. – 17s. – 9d. +.

2'3225) : 273'5208 :: 1. (117'769

(2)
(4)
2

23225
41270
23225
180458
162575
178830
162575
162555
139350
232050

3 = '25
17 = '0708
10 = '0017
'3225

If a pound of any Commodity or Grocery cost 1s. – 1d. – ob. what will 1 C-weight of the same cost? Answer, 6l. — 5s. — 11d. — 3f. +

'00893) . 1'125 :: 1. (125'9809

1786
3572

(1)
(2)
2

893
2320
1786
5340
4465
8750
80371
7230
7144
8860

One payd for his dyet in 3 weekes and 4 dayes 22s. 4d. what is that a yeere? Answer, 16l. – 6s. – 1f.+

```
'0685) . 22'3333 :: 1. (326'034
 1370    2055
 2740     1783
 5480     1370
 (2)       4133
 (4)       4110
  2         02333
             2055
              2783
              2740
```

A Souldier receives for his pay in 3 moneths and 2 dayes, 6l. – 3d. what comes his pay to for a yeere? Answer, 23l. — 10s. — 5d. — ob.

```
'2555) 120'25 :: 1. (470,451.
       10224
        18010
        17892
         011800
          10224
           15660
           15336
           + 3240
```

156¾ French Crownes are bought for 32l. 3s. what is that the Crowne? Answer, 4s.-1d.-ob.

156'25). 643::1. (4'1151
62500
0 18000
0 15625
0 23650
15625
80250
78125
21250

Hitherto the Unite hath falne in the third place, in these questions following it will fall in the 2 place.

How many Scotch Markes (at 13d. ob. the Marke) is there in 175l.-6s? Answer, 3116½—,

1'125). 1::3506. (3116'44

3375
(0) 1310
(3) 1125
3 1850
1125
7250
6750
5000
4500
5000
I 2

How

How many Harpers in 3l. - 6s? Answer, 88.

75). 1 :: 66. (88.
600
600

0
(1)
1

How many Florins ought I to receive for 70l. 15s. 6d. each Florin being estimated at 3s. - 2d? Answer 447.

3'1667). 1 :: 1415'50. (447.
148 820
221 520

(1)
(3)
2

If you worke by Nepaires bones, or if you make up the Multiplees of your Divisor to 9, or so many as you can by memory supply the rest, you neede not set downe the multiplyed Divisor, but substract from the Dividend as in the last example, and in some that follow.

How many yards of Satten will 155l. 18s. buy, at 13s. - 4d. the yard? Answer, 233 yards and $\frac{1}{2}$ a naile.

13'3

$$13'333). \quad 1 :: 3118. \quad (233'15$$

```
 0          4420
(2)         4210
 2          2110
            7770
```

How much white Plate will 152l. 10s buy at — 6d. — ob. the pound? Answer, 3 pound. ounces.

$$100,5416. \quad 1 :: 3050. \quad (30'333$$

```
 0          033800
(1)         036380
 1
```

If you exchange money, so that for 1l. English shall have 1l. - 5s. - 9d. Flemish, how much shall ave for 133l. - 2s. Flemish? Answer, 103l. - 6s. - -ob.

$$25'75). \quad 1 :: 2662. \quad (103l'3398$$

```
 08700
 09750
 10250
 25250
```

Observe that the fourth tearme is pounds ; and Decimalls of a pound , because the second tearme was 1l. and not 20s. therefore the corresponding Fractions of s. d. f. are taken out of the first Table of Decimalls.

Examples where the Rule is wrought both by Multiplication and Division.

If a peece of Velvet being 32 yards cost 32l.-5s.-6d. what will 5 yards and a quarter of that Velvet cost? Answer, 5l. — 5s. — 7d+.

32. 645'5 :: 5'25. 105'5898

5'25
32275
12910
32275
32) 3378'875 (105'5898.
178
188
287
315
270

If 23l. of Pepper cost 30l. 7s. — 9d. what shall I pay for 50l? Answer, 66l. — 1s. — 1d. 3f.

23. 60775 : : 50. 1321'149
50

23) 30387,50 (1321'149.
73
48
27
45
120
280

If 100 yards cost 3l. — 6s. — 8d. what shall 55 yards cost? Answer, 1l. 16s. 8d.

100. 66'6667 : : 55. 36'6667
55
33 33335
333 3335
3666'6685

In these and the like questions, putting them into Dicimalls, you multiply the second, and third, and divide by the first according to the Rule; and must use for speede all the former contractions of Multiplication and Division.

One desires me to take as much Hops as come to 11l. – 10s. out of a bag of 2 C 14l. that cost 27 – 19s. – 6d. what quantity must I have? $\frac{3}{4}$ C. 13l. +

559'5. 2'125 :: 230. '8730
230
4250
6375
559'5) 488'750 ('8739 = $\frac{3}{4}$ C
(2) 41150 '1239
(1) 21850
(1) 51750

What is the $\frac{1}{8}$ of a Ship worth, when the $\frac{1}{2}$ is valued at 1015l. – 40s. Answer, 253l. – 17s. – 6d.

'5. 20310 :: '125. 5077'5
125
020310
040620
101550
5) 2538'750 (5077'5
2
(1)
3

Heere in the last question the $\frac{1}{2}$ and the $\frac{1}{8}$ being put into Decimalls; then you double the product, and strike off an Unite for the Quotient.

If an ounce of Gold be worth 3l. – 14s. – 6d. what are 5l. – 11.oz. worth? An. 249l. – 11s. – 10d. '08

'0833. 74'5 :: 5'9167.
74'5
29533 5
(1) 236668
(4) 414169
3 440'78915 (4991'847
8758
8119
1621
7485
4210
6780

What are 3l. 4.oz. 6½ p. of gold worth at 1d.ob. [the] graine? Answer, 123l. — 10s. — 6d+.

.00017. '125 :: 3'35992
'125
167995
(2) 33599
(5) 67198
3 '00017) '4199875 (2470'5147
79
119
0087
25
80
120

A Goldsmith buyes 9 ounces and 12p. of gold for 36l. 12s. what is that an ounce? An. 3l.-16s.-3d. f.

'8. 732 :: '0833.
_732
9oz. = '75 1666
12p. = '05 2499
'8 5831
0 '8) 60'9756 (76'2895
(1)
1

If an ounce of gold be worth 3l.-11s.-5d. what is a penny weight worth? Answer, 3s. 6d. 3f.
What is a graine worth? Answer, 1d. f.

'0833. 71'5 :: '00416. 3'5707
71'5
2080
(4) 0416
(4) 2912
'0833) '297440 (3'5707
2499
4754
4165
5890
5831
5900

'0833

'0833. 7'15 :: '000173. '1484

```
                 71'5
                 865
                 173
                1211
'0833)   ,0123695 ('1484.
 (5)        4039
 (4)        3332
 (1)        7070
            6664
            4060
```

Note that in the former Multiplication ; because it was intimated by the *Factores* that there should be 7 Decimalls in the product, wee put a Cypher to supply the place.

What

What is due for a pension of 3s. – 5d. weekely, behinde for 3 yeere, 9 moneths, and 10 dayes.

```
'01918. 3'4167 :: 3'7773.
                  3'4167
                  ------
                 37773..      9 = '25
               113319....    10 = '0273
               151092...          -----
(3)             226638.            '7773
(5)             264411
---             ---------
 2            12'90590091 (672'6719
               11508:
               ------
'01918)        13939
 3836          13426
 7672          ------
15344           5130
                3836
                ----
                12940
                11508
                -----
                 14320
                 13939
                 -----
                  03810
                   1918
                   ----
                   2892
```

What is due for a Major for 5 moneths 11 dayes for, his pay, at 33s. – 6d. *per* day? Answer, 277l. 11s. – 1d. 1f+.

'00

'0027. 33'5 :: '4474. 5551'107
33'5
22370 5 = 4167
(1) 13422 11 = 0307
(4) 13422
3 '0027) 14'98790 (5551,107 '4474

Hitherto wee have given Examples for the worʃing the Golden Rule only when the Queſtion is ingle, and without any trouble in the Poſition; now will give ſome few Examples wherein ſome acount muſt be made in the ſetting downe the Queion.

(1) If 100l. weight be worth 12l. what is 985l. worth allowing 4l. weight to every hundred for reat, heere becauſe 104l. is delivered for 100l? It is hus.

104. 12 :: 985. 1'44. Anſwer is 144l.

(2) Two depart from one place, the one Eaſtward, and the other Weſtward, the one travelleth miles a day, the other 5 miles a day, how farre are hey diſtant the ninth day after their departure? Anwer, 72 miles. Heere their firſt dayes diſtance is added together which makes 8, and then it is 1. 8 :: 9. 2.

(3) On

(3) One selleth cloth for 350l. and gaynes after 10l. in the 100l. what was the principall, and what the cleare gaine? Answer, the cleare gaine is 31l.—16s. — 4d. +.

110. 100 : : 350. 318'1818.
350'0000
318'1818
31'8182

And now by making up 318'1818 350, I finde the cleare gaine to be 31'8182 either by Addition or Substraction.

4. There are 30 clothes bought for 70l. are sold for 80l. Now if they had cost 80l. how should they have beene sold to have gained after the same rate? Answer, 91l. – 8s. – 5d.

70l. stock. 10l. gaine : : 80l. (11'4286.
70
For 88l.+11'4286=91'4286 100
70
300
20

Therefore had the Cloathes cost 80l. they should have beene sold for 91'4286.

5. One

5. One sold so much Velvet as yeelded him 600l. ready money, wherein he gained cleare above his principall 60l. what gained he upon every 100 of his principall? Answer, 11l. – 2d. 1f.

Now if 540l. 60 :: 100, 11'1111

(6). 160l. of Cloves wanting 6s. 8d. or '3333 was sold at 7s. – 6, or '375 the pound, what gaine is there upon a hundreth pound stock after that rate? Answer, 12l. – 10s.

You must abate the bare stock '3333 from the stock and gaine, 375 leaveth '0417 for cleare gaine; now therefore,

'3333. '0417 :: 100. 12'5
or, '3333. '375 :: 100. 112'5

And by abating 100 from 112'5 rests 12'5 as before.

7. One (suppose *A*) flying every day 40 miles, is pursued the fourth day after by another (suppose *B*) 50 miles a day: in how many dayes, and after how many miles Travaile will *A* be overtaken? Answ. at the end of 12 dayes; for, *B* rideth 50 miles, *A* but 40, and 50 lesse by 40 is 10 miles, the cleare gaines of one day; but because *A* was fled 3 dayes journey, that is 120 miles before *B* pursued, 1 day, 10m. 1d :: 120. 12 dayes.

Then

Then I say againe, if 1. 50 : : 120. 600 miles tha[t] B must goe before A be overtaken : thus somtime[s] for the difficulty of a question the Golden Rule mu[st] be used more then once , before a full answer b[e] had.

9. Thus have I runne over very many question[s] whereby the excellency of this rule is perceived, th[e] effects whereof are very many, as (1) by the price o[r] value of one thing to finde the value of many , (2) by the price or value of many to find the value of on[e] (3) by the value of many to finde the value of many &c.

10. If you will prove the Rule according to Sect. 3, the Product of the first tearme given , and the la[st] found, ought to be equall to the Product of the se-cond and third.

Of the Rule of Three indirect, or of the inverse Rule of Proportion.

This Rule is called the inverse or backward Rule of Proportion, because it inverts the practice of the former Rule : For in this Rule the first and the second numbers are to be multiplyed , and the product is to be divided by the third Number ; and it is easily knowne whether the Question must be wrought by this rule or no; for if more require lesse, or lesse more, the Question is wrought by this rule, as will more e-vidently appeare by examples. As if 4 Horses in six dayes

[illegible] ayes eate 10 bushells, 8 horses must needs eate 10 bushells in a lesser time, and heere 10 bushells the common tearme is omitted in the figure, and the question stands thus.

4 : 6 : : 8 : 3. 8 horses will eate them in 3 dayes.

If a two penny loafe of bread weighed 6l. – 3oz. when a Bole of Rye cost 6s.-6d. what is a bole of Rye worth when a two penny loafe of bread weigheth but 2l.-4oz? Answer, 17s.-10d. 1f.

```
6'1875 . 6'5 : : 2'25.
             65
       --------
        309375
       371255
       --------
       40'21925

          40'21925 (17'8707
(1)  2'25) 1771
(2)        1969
---        ----
 1          1692
            01750
```

One borrowed money, *viz.* 4000l. for 3 yeeres of his friend, which when he came to restore the debt, his friend would take no consideration, but only desired, that he would lend him a summe another time when he stood in neede, he after lent him 7480l. How long was he to detaine that summe, to requite

the former courtesie? Answer, 1 yeere, 7 moneths, 13 dayes +.

4000. 3 :: 7480. 1'6042

	3
7480)	12000 (1'6042.
14960)	7480
29920)	45200
	44880
	32000
	29920
	20800

Which out of Decimalls by the Table of time is 1y. 7m. 10d.

A City besieged hath victualls to maintaine 5000 Souldiers six moneths; it is desired to know what Souldiers it will maintaine for 9 moneths? Answer, 3333 Souldiers.

6. 5000 :: 9. 3333 +

6
30000 (3333'33

If 20 Pyoners can finish a worke in two moneths and 10 dayes, how many Pyoners will finish that worke in 20 dayes? Answer, 69.

'194. 20 :: '0548) . (69.

20   3'880   2. 1667

3,880   3 288   10. 0273

5920   '1940

How much Plush is necessary to lyne a cloake, which hath in it 4 yards of 1 yard 3 quarters broad when the Plush is but $\frac{3}{4}$ and $\frac{1}{8}$ broad? Answer, 7 yards, 3 quarters, and $\frac{1}{4}$.

1'75. 4 :: '875) . (7'999

7'00

4   6 125

7'00   8750

7875

8750

7875

If for the matting of a roome, there needes 100 yards of yard broad, how many yards will be needfull of 1 $\frac{1}{2}$ broad? Answer, 66 yards $\frac{2}{3}$ +.

1. 100 :: 1'5. 66'666.

Unto how many Florins doe 487 Dollars amount unto, accompting the Dollar to consist of 28 $\frac{1}{2}$ Pattern peece, and the Dollar at 20? Answer, 694 –.

In an Acre of Land, if the breadth be 4 pearches, the length must be 40, now if the breadth be $8\frac{1}{2}$; what length must I have to make the Acre? Answer, $18\frac{3}{4}$ † Of Pearches.

4. 40 :: 8'5. 18'82

4

8'5) 160 (18'82

85

750

680

700

680

200

170

The effects and uses of this rule are very many, as may be gathered out of the aforegoing Examples.

CHAP.

Chap. X.

Of the double Golden Rule, or Compound Rule of five Numbers.

1. THis Rule is called double or compounded, because it is wrought by two workes of the Golden Rule, and is called the Compound Rule of five Numbers, because the given tearmes are alwaies five, wherefore three are conditionall and Antecedents or suppositions, the other two are interrogative, and are consequents, matching, or answering some of the former Antecedents, insomuch that there are as many consequents as Antecedents: one lesse of like kind, or Denomination with the Antecedents.

2. For the right placing the tearmes in this Rule, first in three tearmes, set down the conditionall part, let that which is the principall cause of losse or gaine, Interest or Decrease, Action or Passion be put in the first place, that whose Sirname betokeneth the space, of time, distance of place &c. be put in the second, and the other remayning be put in the third place; then the conditionall part being rightly placed, the two tearmes wherein the demand lyeth,

 must

must correspond to some of the conditionall tearmes, and be placed under them.

3. This Rule is likewise direct or converse, but lest the Reader should be troubled with too many distinctions, take this one Rule for both; after you have placed the Question by Section 2, then if the tearme sought fall under the third tearme, the Rule is direct, and thus to be wrought. (1) *Multiply the three last tearmes for a Dividend, and the two first for a Divisor, finish Division heereby, and the Quotient is the tearme sought.* (2) But if the tearme sought fall under the first or second Tearme of the conditionall part, then the Rule is, *Multiply the first, second, and last, for Dividend, and the third and fourth for Divisor, finish Division heereby, and the Quotient will give the tearme that is wanting*; Examples will make the Rules more perspicuous.

The Posture, and excellent easie performance of this Rule I received from the aforementioned Nicholas Shuttleworth *of Foret Esquire, the Paragon of learning and virtue.*

(1) If 12 rood of ditching be wrought by two men in 6 daies, how many Rood shall be wrought by 8 men in 24 dayes: according to section 2 I place 2 for men in the first place, who are the principall cause, 6 dayes in the second, 12 in the third, and their match tearmes under them; which done, I perceive the blanck or tearme unresolved falls under the third tearme, therefore according to the first rule,

I multiply the three last tearmes for Dividend, and the two first for Divisor, which done the Quotient gives me 192 for the sixth tearme or resolution of the Question.

Men	Dayes	Rood
2.	6.	12.
8.	24.	

$$12 \times 8 \times 24 = 2304\ (192$$
$$2 \times 6 = 12)\ \underline{12}$$
$$110$$
$$\underline{108}$$
$$24$$

(2) If 12 Rood require 2 men to worke in it 6 dayes, how many men will 192 Rood require to worke it in 24 dayes? Heere the blanck falls in the first place, and therefore by the second Rule the answer is 8 men.

Men	Dayes	Rood
2.	6.	12.
	24.	192.

$$2 \times 6 \times 192 = 2304\ (8$$
$$12 \times 24 = 288)\ \underline{2304}$$

3. If 2 men worke 12 rood in 6 dayes, in how many dayes will 8 men worke 192 Roods? Heere the blanck falls under the second tearme of time, and

and therefore anſwered by the 2 Rule which gives the dayes to be 24.

Men	Dayes	Rood	
2.	6.	12.	2* 6* 192 = 2304 (24
8.		192.	12* 8 = 96. 192
			384
			384

So in this Example of ſimple intereſt.

1. If 100l. in 12 moneths gaine 10l. what ſhall 50l. gaine in 6 moneths ? Anſwer by the firſt Rule, 2l. — 10s.

l.	m.	l.	
100.	12.	10. }	10* 50* 6 = 3000 (2'5
50.	6.	}	1000* 12 = 1200)

2. If 10l. come of a hundreth in 12 moneths, what ſhall come of 50 ſhilling gaine in 6 moneths ? Anſwer by the ſecond Rule, 50l.

l.	m.	l.	
100.	12.	10. }	100* 12* 2'5 = 300|0 (50
	6.	2'5. }	10* 6 = 60) 300

3. If

3. If in 12 moneths 100l. give 10l. in how many moneths shall 50l. give 2l.10s.? Answer by the second, 6 moneths.

l.	m.	l.		
100.	12.	10.	100 × 12 × 2'5 = 3000.	(6
50.		2'5	10 × 50 = 500)	3000

And after this manner may any question resolvable by the two first parts of this Rule be performed.

4. But if 4 tearmes of explanation or conditionality precede the tearme of the Question; as how

Many { Of the first will make some number of the last sirname, the Question is answerable by the Compound Rule *discending.* Of the last named will countervaile some certaine number of the first named, the question is answerable by the Compound Rule *ascending.* }

The worke of the Compound Rule discending.

Multiply the first, second, and fifth tearmes for the Dividend, and multiply the second and fourth for the Divisor, finish Division *thereby, the Quotient gives the answer.*

Example, If 2 Angels countervaile 20s. and 12s. countervaile 2 Crownes; how many Angels will

will countervaile 10 Crownes ? Answer, 6 Angels

2an.	.2os.	$2\times 12\times 10 = 240$ (6
12 s.	.2cr.	$20\times 2 = 40)$ 240
10cr.	.6an.	

The worke of the Compound Rule ascending.

Multiply the 2. 4. and fifth tearmes together for the Dividend, and multiply the 1 and 3 for the Divisor, finish Division &c.

Examp. If 2 Angells countervaile 20s. and 12s countervaile 2 Crownes, how many Crownes will countervaile 6 Angels? Answer, 10 Crownes.

2an.	.20s.	$20\times 2\times 6 = 240$ (10
12s.	.2cr.	$2\times 12 = 24)$ 24
6an.	.10cr.	00

5. If any question in any of these rules have like numbers in the Dividend, and Divisor, omit, or cancel them to save labour, and worke onely with the whole numbers.

The use and effects of these foure precedent rules are of great moment; for upon the first two, not only all questions of Merchandize, wherein time is bought and sould, but also by them all charges of Warre, touching victualls, Souldiers wages, expence of powder, casting of Trenches, and other

Militar

Military matters are ſpeedily, and aptly diſcuſſed; and upon the later two of them, depend the equation of divers Barters, with the exchange and reduction of all manner of moneys and coynes, weights and meaſures of what nature ſoever, as may appeare by the queſtions following; wherein leſt we may ſeeme too tedious: the operations are omitted.

Queſtions for practiſe of the precedent Rules.

1. If 30 buſhells of ſeed will yeeld in one yeare 60 buſhells, how many will 80 buſhells yeeld in 7 yeares? Anſ. 6720.

2 If 6 Mowers will mow 45 Acres in 5 dayes, how many Mowers will mow 390 Acres in 6 daies? Anſ. 33·666.

3. If in ten dayes of 12 houres long, a man may journey 300 miles, in how many dayes of 16 houres long may he travell 500 myles? Anſwer 12·5.

4. If 12 penny worth of wine ſatisfie 8 perſons at a meale, when wine is at 6d. the quart, how many ſuch perſons will 20 penny worth of wine ſatisfie, when wine is at 4d. the quart? Anſwer 10.

5. If 2 Plowes till 18 Acres in 6 dayes, how many Plowes will car 45 Acres in 3 dayes? Anſwer 10.

6. If 500 Pyoners caſt a trench of 300 Rood long in 6 houres, how many Pyoners will caſt a trench of 160 Rood in 2 houres? Anſwer 800.

7. If 15 ells of ſtuffe $\frac{3}{4}$ or ·75 Quarters broad coſt

37s.

37s. – 6d. what shall 40 ells of like stuffe cost, being $\frac{7}{3}$ or 1'66 broad? Answer 11l. 4s. +.

8. One buyeth stuffe for 22s. 6d. the peece ready money, and would sell it againe for 24s. the peece, what time may he forbeare his money, and yet gaine after 9l. in the 100 for 12 moneths? Answ. 8 moneths 8 dayes.

9. One buyeth 20 yards of satten for 12l. 10s. ready money, for how much more then it cost may he sell the yards giving two moneths day for payment to gaine after 10l. in the hundreth for 12 moneths?

For answer, because the position is of 20 yards, and the question but what he may raise in the sale of one yard only, and not in the whole 20 yards, I first learne the price of one yard thus; 20 yards, 12'5l: : 1. '625. Then having the prize of one yard, I place the Question, by the first Rule, and the answer is 14s. – 5d. +.

10. A Towne is beseiged wherein is 3000 Souldiers, who have sufficient victualls for two moneths; but they looke for no ayde to raise the seige till 6 moneths; how many Souldiers may the Captaine dismisse to make the victualls serve so long? Answer, he must dismisse 2000.

11. If 5 Cannons in two dayes spend 60 barrells of powder, what will 14 Cannons spend in 5 dayes? Answer 420.

12. If one horseman for one moneths wages hav

what treasure will pay the wages of 4000 horse-
n 9 moneths? Answer, 108000l.
13. If 8 Tablers when wine is at 6d. the quart
nd every meale at their ordinary 12d. in wine,
ien wine is falne to 4d. the quart, what shall they
n spend a meale, not bating their usuall quantity?
For answer, though the men seeme to be the chiefe
use of the spending of the wine; yet because their
mber remaineth alwaies one, and they drinke no
re at one time then another; the cause therefore
he increase or decrease of this expence, is rather
be attributed to the price of the quart of wine,
ose alteration is the chiefe cause of the increase
decrease of their charges; Therefore according
the Rule the answer will be 8d.
14. If 4 penny worth of bread suffice 12 persons
meale, when wheate is at 20s. the quarter? How
ny penny worth will suffice, when wheate is at
. the quarter? Answer 3d.
15. If 15 Alnuds French, make 100 sticks Fle-
sh, and 100 sticks Flemish make 60 English ells;
w many Alnuds is 150 English ells? Answer by
third Rule 28.
16. At Roan one *A* delivered *B* in exchange 100
ancks, every Franck there worth 50 Soulz Tourne,
upon condition to receive for the same at *London*
er 4s. – 11d. for every Crowne of 50 Soulz
urne; how much sterling money will pay the
l of Exchange at *London*? For answer, frame the
question

question thus ; If 1 Francke countervalie 20 Soulz and 500 Soulz countervaile 4s. 10d. or 4s.8333. How many pence sterling is 100 Francks? Answer, 232 pence.

17. If 4d. *Paris* be worth 5d. *Tournois*, and 10d. *Tournois* be worth 12d. of *Savoy*, how many pence of *Savoy* are 15 pence *Paris*? Answer $22\frac{1}{2}$ of *Savoy*.

18. If 100l. weight of *Perose* be worth 90l. weight of *Sene*, and 100 weight of *Sene* be worth 120l. weight of *Pyse*; how many pounds weight of *Perose* will weigh 324 pound weight of *Pyse*? Answer 300.

19. If 4 ells of *Antwerpe* measure make 3 yards of *London* measure, and 15 yards of *London* measure containe 12 ells of *Lyon* measure; how many ells of *Antwerpe* measure is contained in 60 ells of *Lyon* measure? Answer 100.

20. If 35 ells of *Vienna* make 24 at *Lyons*, and 3 ells of *Lyons* 5 ells of *Antwerpe*, and 100 ells of *Antwerpe* 125 ells at *Franckfurt*; how many ells of *Frankfurt* make $42\frac{3}{4}$ ells at *Vienna*? Answer $60\frac{1}{4}$.

21. Thus have wee handled the Rule of Proportion in all his parts, which who perfectly understands, may worke any Arithmeticall question wherein a proportion is given; yet sometimes questions require iteration of this Rule, the most whereof are comprehended in these two following, *viz.* the Rules of *Fellowship* and *Alligation*.

CHAP.

CHAP. VI.

The Rule of Partnerſhip or Fellowſhip.

THis Rule is of great uſe to ballance Accounts, when many put together a generall ſtock, ſo that every man muſt have his proportionable part of his gaine: for as the whole ſtock (or generall Antecedent) is to the totall ereby gained or loſt (which is the generall conſe-ent) ſo each mans particular ſhare, is to his proper are of loſſe or gaine; therefore that the queſtion ay be rightly ſtated; the ſeverall monies of all the verall Partners muſt be gathered into one ſumme hich muſt be placed for the firſt tearme in the Gol-n Rule, the common Gaine or loſſe muſt be ſet in e ſecond, and in the third place muſt be placed eve- mans particular ſtocke, working the Golden Rule many ſeverall times, as there were partners at joyned in the worke.

2. Of this Rule there are two kindes. (1) The ſt kinde doth ſeverally expreſſe the third tearme or ery mans particular ſtock, as in this Example.

A and *B* buy a Tunne of wine for 20l. *A* layd out 12l. *B* 8l. and they gained in the saile 5l. cleerely; how much shall each have of the gaine?

Answer, *A* 3 } And 3+2=5 the whole gaine *B* 2 } *A* 20. 5 :: 12. 3 *B* 20. 5 :: 8. 2.

3. The second is when every particular stock is found out, sometimes by Multiplication, and so likewise the totall of the Products is the generall consequent, and sometimes the totall of all such products is a generall Antecedent, in the Golden Rule to the totall or generall consequent.

Questions resolved by the first part.

1. *A. B. C.* Frought a Ship with 70 Tun of wine, whereof *A* laded 30, *B* 24, and *C* 16 Tun: by extremity of tempest 10 Tunne of wine is cast over board; how much of this losse must each of these Merchants beare?

Answer, 30+ 24+ 16 = 70l. the totall stock. 10l. the totall losse, whereof *A*. 4'2857. *B*. 3'4285. *C*. 2'2858.

A. 70. 10 :: 30. 4'2857. *B*. 70. 10 :: 24. 3'4285. *C*. 70. 10 :: 16. 2'2858. and 4'2857+ 3'4285+ 2'2858 = 10.

(2) On

(2) One dyeth indebted to 8 severall persons, viz. to *A, B, C, D, E, F G,* and *H* to *A* 100l. to *B* 86, to *C* 75, to *D* 60l. to *E* 54l. to *F* 32l. to *G* 24, and to *H* 15l. all his goods will but amount to 150l. whereof the Creditors agree to take their proportionable parts; what shall each Creditors share be, the summe of their severall debts being 446l? Answer, by working as aforesaid: *A* must have 33'5323, *B.* 28'92385. *C* 25'2242. *D.* 20'1802. *E* 18'1514. *F* 10'7623. *G.* 8'0517. *H* 5.'4448. which while summes and fractions being added together, according to the Rule afore taught account to the summe of 150l. the whole value of the goods aforesaid.

For: 446. 150 :: 100. 33'5323 :: 86. 28'9238 :: 75. 25'2242 :: 60. 20'180254. 18'1514 :: 32. 10'7623 :: 24. 8'0517 :: 15. 5'0448.

(3) *A. B* and *C* had a common stock of 500l. and lost thereof 124l. *A* his losse was 20l. *B.* 43l. *C* 61l. what was every mans stock at the first? Answer, the whole losse being 124l. therefore 20l. + 43l. 61l. = 124. therefore *A* his stock was 80'6453, *B* 173,3870, and *C* 245'9677. For: 124. 500 :: 20. 80'6453 :: 43. 173'3870. 61. 245'9677. and 80'6453 + 173'3870 + 245'9670 = 500 the stock.

The proofe of these and the like questions is found by adding up into one summe, all the particular summes found by the Golden Rule being the Part-

 ners

ners respective losse or gaine, which if it agree with the first generall consequent ; the question is well proposed and wrought.

Therefore divers questions impossible to be answered may be propounded, as when either the parts exceede the totall or the Partners want of the due shares appoynted by the Question ; and therefore it is best alwayes to see that the Partners shares agree with the generall Antecedent, for then consequently their other parts must agree with the generall consequent.

Questions resolved by the second Part.

1. There is an Army of 6000 foote, and 800 horsemen, the stipend of each footman is 4 shillings a weeke, and of every horseman is 9s. a weeke, they are to divide a booty of 2000s. so as their particular parts may be proportionall to their wages, how much belongeth to the footmen, and how much to the horsemen? Answer, horsemen must have 461'54, that is 23l. - 1s. - 6 $\frac{1}{2}$ footmen 1538'46, that is 76l. - 18 - 5 $\frac{1}{2}$.

The worke.

800×9 = 7200	*The particular later Antecedents.*
6000×4 = 24000	
31200	*The first or generall Antecedent.*
2000	*The generall consequent.*

Therefore

Therefore 31200. 2000 : : 24000. 1538'46 : : 7200. 461'54.

2. *A. B* and *C* take a pasture for 30l. a yeere Rent, wherein *A* feedeth 20 Oxen 70 dayes, *B* feedeth 46 Oxen 56 dayes, *C* feedeth 32 Oxen 60 daies; what shall each pay towards the Rent in proportion to his time? Answer, *A* must pay 7'1234. *B* 13'1072. *C* 9'7693.

The worke.

20×70 = 1400 *Therefore.* 5896. 30 : : 1400. 7'1234 : : 2576. 13'1072 : : 1920. 9'7693.
46×56 = 2576
32×60 = 1920
5896 *And* 7'1234 + 13'1072 + 9'7693 = 30.

3. A Colledge hath 20 Fellowes, and 30 Scholars, which may dispend yeerely 2600l. but every Fellow hath 5l. for every Scholars one: How much of this Revenue belongs to the Fellowes, and how much to the Scholars? Answer, Fellowes 2000l. Scholars 600l.

The worke.

20×50 = 100 *Therefore.* 130. 2600 : : 100. 2000 : : 30. 600.
30×1 = 30
130 *And* 2000 + 600 = 2600.

4. *A. B. C* and *D* make a ſtock to traffick with a yeere, *A* diſburſed 600l. for 10 monthes. *B* 400l. for 8 monthes, *B* 800l. for 6 monthes, *D* 500l. for 12 monthes; they gaine 760l. clearely, what portion ought each to have? Anſwer, *A* gained 228l. *B* 121'6. *C* 182'4. *D*. 228l.

The Worke.

600×10 = 6000	*Therefore.* 20000. 760 : : 6000.
400×8 = 3200	228 : : 3200. 121'6 : : 4800.
800×6 = 4800	182'4 : : 6000. 228.
600×11 = 6000	*And* 228+121'6+182'4+ 228 =
20000	760.

5. *A. B.* and *C.* enter Partnerſhip upon the firſt of *January* for a whole yeere, *A* the ſame day diſburſed 100l. whereof he received back againe upon the firſt of *Aprill* 20l. *B* delivereth the firſt of *March* 60l. and more the firſt of *Auguſt* 100l. *C* layeth out 140l. the firſt of *July*, but the firſt of *October*, withdraweth 40l. at the yeeres end their cleare gaine is 142l. how much thereof ought every particular Partner to have? Anſwer, *A.* 51l. *B.* 55l. *C.* 36.

The Worke.

100×3+ 80×9 = 1020	*The particular Antecedents.*
60×10+ 100×5 = 1100	
140×3+ 100×3 = 720	
2840	*The firſt Antecedent.*

Therefore

Therefore. 2840. 142 :: 1020. 51 :: 1100. 55 :: 720. 36.

And 51+55+36 = 142.

In the last and like questions the particular Antecedents are gotten by multiplying their severall disbursments by the severall time, and because within the whole time they tooke out and put in againe, therefore after you have multiplied the summes put in, you adde them together, as in the last, because *A* had 100l. betwixt the first of *January*, and the first of *March* which is 3 Monthes, and then 80l. 9 monthes longer; after you have multiplyed 100×3 which is 300, and 80×9 which is 720, you adde them together, and it makes 1020, and so of the rest.

6. *A. B. C* and D partners for two yeeres, make their stock thus. *A* put in presently 30l. but at 8 months end received 10l. back, for which in the beginning of the 20 moneth he restored 12l. *B* put in presently 24l. but at 6 monthes end tooke back 8l. for which in the beginning of the 16 moneth he put in 14l. *C* put in 20l. presently, but at 7 monthes end received it whole againe, yet at the beginning of the 18 moneth, restored 14l. D put in nothing untill the beginning of the 7 moneth, and then put in 18l. but after 4 monthes received back 9l. for which in the beginning of the 17 moneth he restored 15l. Now at the 2 yeeres end, they had gayned 100l. how much thereof ought each Partner to have? Answer,

A 35'469. B 31'9222. C 14'4264. D 18'1924.

The worke.

30×8+20×16+12×5 = 620	Therefore. 1748. 100 ::
24×6+16×18+14×9 = 558	620. 35'469. 558.
20×7+16× 7 = : . 252	31'9222 : : 252.
18×4+9 ×14+15×8 = 318	14'4264 : : 318.
1748	18'1924

And 35'469+ 31'9222+ 14'4264+ 18'1924 = 100.

The accounting of the partners times of putting in and taking out of their stocks, will be little trouble; if you first regard the time agreed upon for continuance of the Partnership; and then how many monthes every partner suffereth his proper share to continue in banck.

The summe may as easily be wrought if the stock remaine not for whole monthes, as in the question following for dayes.

7. Two Grasiers A and B tooke a pasture for 40l. per Annum, viz. 800s. they both put in at the first day of January 50 a peece; but A the 25 day of March put in 20 more, and the third day of May tooke 22, and put in the 13 of September 7 more; B tooke out the second day of February 9, and put in the 10 of June 32; how much must each pay of the rent? Answer, A must pay 451'64, that is 22l. 11s.

118 – 6 $\frac{1}{2}$ and *B* 348'36 that is 17 – 8 – 5 $\frac{1}{2}$.

The works.

$$50 \times 84 + 70 \times 39 + 48 \times 133 + 55 \times 109 = 19307$$
$$50 \times 33 + 41 \times 128 + 73 \times 204 = \;.\;.\;.\;.\; \underline{14892}.$$
$$34199$$

Therefore. 34199. 800 : : 19307. 451'640 : : 14892. 348'360.

And 451'640 + 348'360 = 800.

If the queſtion be of broken parts then wee put them into Decimalls, and work them as before.

CHAP. XII.

Of Alligation.

1. THis Rule is very neceſſary to mix quantities of ſeverall Rates, and to diſcover the mean price, as alſo in compoſitions of medecines, for both the quantity and price; Heerein wee conſider the generall *Propriety* of the Rule, the ſpecies which are two, viz. Alligation Alternate and Mediall.

 4. The

2. The generall obſervation to be had is. 1. That all the ſeverall quantities and meaſures may be *Homogeneall*, that is alike, and of the ſame kinde. 2. The price of the mixed quantity is a meane price betwixt ſome of the prices of the ſimple quantities.

3. Alligation alternate, doth alter or change the *Location* of the exceſſes or differences falling out betweene the meane price of both the extreames, aſcribing that to the greater extreame, which proceedes from the leſſer *& contra*, and this being done, the ſaid exceſſes ſo alternally and interchangeably ſcituated, doe forthwith declare the due proportion of every ſimple, entring the mixture; in the alligating the extreames of the prizes you are to obſerve theſe three Rules.

.1 That each greater extreames muſt be linked with the leſſe; the thing whereof the Rule taketh his name, As

2. That when both the greater and leſſer extreames are not ſingle, that then they may be linked ſo diverſly and ſo often, that ſundry exceſſes and differences may be applyed for one, and there will be diverſity of anſweres, and yet all true, As

3. If either of the extreames be ſingle, and the other extreames be plurall, the ſingle extreame muſt be

be linked with all the rest, and then there can be but one answer.

4. Of Alligation alternate there are these three varieties.

1. If the price of every of the simples be expressed; but no quantity, and it be demanded how I may mix the simples to sell one measure or quantity at a meane rate, then meere Alligation alternate answereth such questions.

2. If the measure or quantity of one of the simples, and the prizes of every of the simples be expressed, and it be demanded how much or what quantity of every of the other simples may I commix with the simple expressed to sell at a meane rate, then Alligation alternate, and the golden Rule performe such Questions.

3. If the price of every simple, but none of their quantities be expressed, and it be demanded how much must be taken to make up a certaine quantity to be sold at a meane rate, Alligation alternate, addition of the excesses, and the Golden Rule performe such questions.

Examples of the first variety.

1. There is Wheate at 28d. the bushell, Rye at 20d. Barley at 14d. and Oates at 10d. how may a mixture of these be made to be sold at 16d. the bushell?

16	28	6 Wheat.
	20	2 Rye.
	14	4 Barley.
	10	12 Oats.
		24

Answer, there must be taken 6 of Wheat, 2 of Rye, 4 of Barley, and 12 of Oates to sell a bushell of that mixture at 16d. the bushell: According to Sect. 3. ob. 1. I linck the extreame 28 with 10, and 20 with 14. and the excesse betwixt 28 and 16 I set 10, and so of the rest.

The proofe of this question, and the like of this sort is; that if the summe of all the quantities found multiplyed in the meane price propounded, its product, be equall to the summe of the severall Products of each quantity in his rate or price: As in the last example, the summe of the quantities found is 24. Now $24\times16 = 384$ and $28\times6 = 168$. $20\times2 = 40$. $14\times4 = 56$. $10\times12 = 120$ And. $168+40+56+120 = 384$. as before.

According to Sect. 3. ob. 2. the extreames being plurall, *viz.* 14 and 10, lesse then 16 and 28 and 20 greater, the Alligation may be altered, and yet the answer true; thus

	d.	br.
d	28	2 Wheat.
16	20	6 Rye.
	14	12 Barley.
	10	4 Oates.
		24

And so by this Alligation there must be taken 2 of Wheat, 6 of Rye, 12 of Barley, and 4 of Oates, to sell a bushell at 16d. The proofe the same

...me for 384 = the Products of 28×2+ 20×6+ 14×2+ 10×4.

According to Sect. 3. ob. 3. If the meane price given had beene 12d. then the Alligation would be full, and the answer thus;

	d.	b.
	28	2 Wheat.
12	20	2 Rye.
	14	2 Barley.
	10	16. 8. 2.
		24

You must take 2 bushells of Wheat, 2 of Rye, 2 of Barley, and 26 of Oates to sell a bushell of that mixture at 12.

The proofe, 32×12 = 384 and 28×2 = 56. 20×2 = 40. 14×2 28. 10×26 = 260 and 56+40+28+260 = 384, as before.

2. I would know what quantity of Sugar at 14d. the pound, Carrow-seedes at 3s. or 36d. the pound, egges at 1d. the pound, and flower at 2d to make a pound at 15d.

	36	14+13+1.	28l. Carrowway-seede.
15	14	21	21 Sugar.
	2	21	21 Flower. &c.
	1	21	21 Egges.

Examples of the second variety.

1. I have 10 bushells of Wheat at 28d. the bushell,

ell, to mixe with Rye at 20d, Barley at 14d. and Oates at 10d. how many buſhells of thoſe other ſorts ſhould I commix therewith , that the price of the mixture be afforded at 16d the buſhell; and yet juſtly countervaile the price of the ſimples?

16	18	6 Wheat.
	20	2 Rye.
	14	4 Barley.
	10	12 Oats.

Then if 6 buſhells Wheat. 2 Rye : : 10 wh. 3'333

6. 4 : : 10. 6'666

6. 12 : : 10. 20

Therefore I conclude that for every 10 buſhells of wheat I muſt take $3\frac{1}{3}$ + of Rye $6\frac{2}{3}$ + of Barley, and 20 of Oates at the ſame prices, and afford the mixture at 16d. the buſhell: For

I ſay,
- 10 buſhells of wheat at 2'3333 = 23'3333
- 3'333 buſhels of Rye at 1'6666 = 5'5561
- 6'6667 of Barley at 1'16667 = 7'777
- 20 buſhells of Oates at '8333 = 16'666

And 40 buſhells at 16d. the buſhell = 53'2331

The ſame worke will hold in all the ſeverall obſervations of linking the values as in the firſt varieties, and yet all true.

2. I have 100 quarts of Canary at 12d. the quart to

be mixed with Sherry at 9d. & Maligo at 6d. how many quarts of Sherry and Maligo must I mix with the 100 quarts of Canary to sell it at 10d. the quart?

10	12	4+1 = 5	Then 5. 2 :: 100. 40.
	9	2 = 2	5. 2 :: 100. 40.
	6	2 = 2	

Therefore I conclude that for every 100 quarts of Canary I must take 40 of Sherry, and 40 of Maligo to sell it at 10d. the quart in mixture.

And for proofe say.

100 Quarts of Canary at 12d.	=	100s.
40 Quarts of Sherry at '75	=	30.
40 Quarts of Maligo at '5	=	20
And 180 Quarts of the mixture at '8333	=	150'000

Questions of the third variety.

1. I have wheat of 20d. the bushell, Rye of 16d. Barley of 8d. and Oates of 6d. the bushell, and would mix 100 bushells, so as I might afford it at 10d. *per* bushell; how much of every sort must I take?

	d.	b.
10	20	4 Wheat.
	16	2 Rye.
	8	6 Barley.
	6	10 Oates.

Then

Then 22. 100 :: 4. 18'1818 = 18 $\frac{2}{11}$ bushels of wh[eat]
say. 22. 100 :: 2. 9'0909 = 9 $\frac{1}{11}$ of Rye.
22. 100 :: 6. 27'7272 = 27 $\frac{8}{11}$ of Barley.
22. 100 :: 10. 45'4545 = 45 $\frac{5}{11}$ of Oates.

Mix gold of 20. 22 and 24 Carracts fine with Alloy so that 100 ounces of that mixture may be but 18 Carracts fine?

18	20	18	=	18
	22	18	=	18
	24	18	=	18
	0	2+4+6	=	12
				66

Then say 66. 100 :: 18. 27'2727 } Therefore
66. 100 :: 12. 18'1818 } mix 27 $\frac{3}{11}$ of each of the Golds, with 18 $\frac{2}{11}$ ounces of Alloy, it will be 18 Carracts fine.

3. Abate Silver Bullion of 9 $\frac{1}{2}$ ounces fine unto 6. oz. how much Copper Alloy must be mixed, and how much of the Bullion to make 100 weight of the finenesse required?

6	9 $\frac{1}{2}$	6 Of Silver.	}
	0	3'5 Of Alloy.	}

I conclude for every 6 ounces of silver I must take 3 and $\frac{1}{2}$ ounces of Alloy.

4. Com-

4. Compound silver of 3. 5. 8 and 10 ounces wh[...]e to be just 6 ounces fine.

3	4	Therefore I conclude that I must take 4 ounces of 3 fine, and 2 of 5, and 1 of 8, and 3 of 10 to make it 6 ounces fine?
5	2	
8	1	
10	3	

5. Compound 500l. weight of Silver out of [B]ullion of 3. 5. 8. and 10 ounces fine, that it may be [ju]st 6 ounces fine?

3	4	Then say	10. 500 :: 4. 200.
5	2		10. 500 :: 2. 100.
8	1		10. 500 :: 1. 50.
10	3		10. 500 :: 3. 150.
	10		

[20]0. oz. of 3. oz. fine.
[10]0 — of 5. oz. fine.
[5]0 — of 8. oz. fine.
[15]0 — of 10. oz. fine.

Therefore I conclude that must make up the 500l. weight at 6 ounces fine.

6. A certaine Vintner having divers sorts of [W]ines, *viz.* the best at 5s. 3d. the Gallon, others at [.] - 6d. the Gallon, othersome at 2s. the Gallon, [an]d some at 1s. - 6. he is desirous to sophisticate the [W]ine, and fill a Hogshead containing 100 Gallons [wi]th the mixture of these wines, which he may af[fo]rd at 2s. 3d. the Gallon.

2.3	5.3	0.	9d.
	3.6	0.	3
	2.0	1.	3
	1.6	3.	0
		5.	3

Then I say. 5'25. 100 :: '75. 14'28
5'25. 100 :: '25. 4'78
5'25. 100 :: 1'25. 23'80
5'25. 100 :: 3. 57'14

Therefore I conclude that the Vintner must take 14'28 Gallons of the 5s. 3d. *per* Gallon, 4'78 Gallons of the 3s. 6d. *per* Gallon, 23'80 of the 2s. *per* Gallon, and 57'14 of the 1s. 6d. *per* Gallon to make up 100 Gallons at 2s. – 3d.

If the Vintner desire to take yet more of some then another of the wines, he may linke the prizes as in 3 observations of section 3 heereof, and yet satisfie the quantity.

7. A Goldsmith hath divers sorts of silver, *viz.* some of 14 ounces, and 12 penny weight fine, other some of 10 ounces, another sort of 9 ounces, 11 penny weight fine; and some of 8 ounces 10 penny weight; he desires to produce a masse of silver weighing 30 ounces, 6p. and bearing 6 ounces, 10 penny weight, and 6 graines fine; how much ought he to take?

Th-

The worke.

	oz	p	gr
	14.	12.	0.
—	6.	10.	6.
	8.	1.	18

	oz	p	gr
	10.	0.	0.
—	6.	10.	6.
	3.	9.	18.

	oz	p	gr
	9.	11.	0.
—	6.	10.	6.
	3.	00.	18

	oz	p	gr
	8.	10.	0.
—	6.	10.	6.
	1.	19	— 18

I now am to linck them together, but I perceive that the given rate being lesse then any finenesse propounded, I must mix some alloy among to satisfie the demand, the which being alwayes of no value, for it at 0.

	oz	p	gr
	14	12.	0
	10	0.	0
6 oz, 10p. 6gr.	9.	11.	0
	8.	10.	0
	0.	0	0

16.	12.	0.	
6.	10.	6.	
6.	10.	6.	The summe is 41.oz. 13p.ogr.
6.	10.	6.	
6.	10.	8.	
8.	1.	18	+3.oz. 9p.18gr, +3.oz. 0p.18gr,
+3.	9.	18	+1.oz. 19p. 18gr.
+3.	0.	18	
+1.	19.	18	
10.	12.	0	

Then by the Golden Rule I say if 41.oz. 13p. 0gr. 30.oz. 6p : : 6. 10. 60. and what shall 16. 12. give?

The worke.

3'4708. 2.525 : : 13833. 1.0063 that is 12.oz. 1p. 12gr.

3'4708. 2'525 : : '5426. 3659. that is 4.oz. 7p. 18.

1'383

1'3833
2'525 Divisor.
691.65 3'4708
2766.5
69165.
27666.
3'49283 (1'0063.
3'4708

220300
208248

120520

'5426 The Multiples of the Divisor.
2'525

27130 3'4708 - 1
10852 69416 - 2
27130 138832 - 4
10852 277664 - 8
1'3700650 ('3649.
104124
228825
208248
205770
173540
322300

Therefore I conclude that the Goldsmith ought to

 take

take of every of the sorts of silver, 4.oz. 7p. 18gr. and of Alloy or Copper 12.oz. 1p. 12gr. to make up the Masse of 30.oz. 6p. at 6.oz. 10p. 6gr. fine.

5. Hitherto hath beene spoken of Alligation alternate; but if the Question be of the price or worth of some part of the mixture, the quantity and price of every simple being knowne, then Alligation alternate is not to be used, but by adding and multiplying, and by the helpe of the Golden Rule such Questions are resolved, which is usually called *Alligation Mediall.*

Examples.

1. Ten bushell of wheat of 16d. the bushell is to be mixed with 18 bushells of rye at 10d. the bushell, what shall one bushell of this mixture be worth? Answer, 1s, – 1f. –

The worke.

10+18 = 28 Then if 28. 340 : : 1. 12'143. The
×16× 10 proofe if 1. 12'143 : : 28. 340.
160+180 = 340

2. There are thirty ells of Holland at 5s, the ell, 35 ells at 7s, 6d, and 40 ells of 9s the ell, and 60 ells of 12s, 6d, the ell; it is demanded how these may be sold one with another, and what price must be had one with another for an ell, to be gainer nor loser.

r above the former price? Answer, 9s, 2d, ob.

Ells. 30 +35 +40 +60 = 165.
Price. ×5 ×7'5 ×9 ×12'5
150+ 262'5+360+750 = 1522'5.

165). 1522'5 : : 1. (9'227
1485
375
450
1200

A Goldsmith having 8l, of silver Bullion of 7,oz, e, 15l, of 8½ oz, fine, and 13l, of 10,oz, fine, melg all together, would know what finenesse 1l, ight of the whole Masse would be? Answer 8½.

Pounds, — 5 +15 +13 = 36
Finenesse, ×7 ×8'5 ×10
56+127'5 130 = 313'5

Then. 36) . 313'5 : : 1. (8'7083
288
255
300
120

For proofe I say, { 8l, of Bullion of 7, oz, fine = 56
15 at $8\frac{1}{2}$ fine = - - - - $127\frac{1}{2}$
13 at 10 fine = - - - - 130.

And the value of 36l, at 8'7084 = . 313'5, oz, fine.

4. A Goldsmith hath melted 12l, 5, oz, of Gold Bullion of 18 Carracks fine, which foure pound, 4 oz, and $\frac{1}{2}$ of 21 Carracts fine, Of how many Carracts fine is 1l, of this Masse? Answer, 18 Carracts, and $\frac{3}{4}$ +

l, 12'4167 + 4'375 = 16'7917
* 18 * 21
222'7096 + 91'875 = 314'5756

Then. 16'7917. 314'5756 :: 1. (18'78
167917
1466586

5. A Mint Master hath a 100l, weight of Gold of 22 Carracts fine, and 20, weight of 19 Carracts fine; and he would so mix them together, that every pound of the said mixture might be 20 Carracts fine, whether doth he neede to mix any Alloy with it, and how much? For answer I must first see what finenesse 1l, of this Masse will be of, when both sorts of this Gold is mixed together, thus.

Pound

ound weight 100 + 20 = 120 Then 120.
× 22 19 2580 : : 1.
2200 + 380 = 2580 21'5, so I finde that one ound of this Masse will be 21 $\frac{1}{2}$ Carracts fine, and herefore it is evident that there must be some Alloy ut to it to abate it from 21 $\frac{1}{2}$ to 20 Carracts fine, hich is found by Alligation alternate, thus.

hus, 20 { 21'5 } { 20. / 0 } 1'5

And therefore for every twenty pounds of Gold, there must be 1 $\frac{1}{2}$ pound of Alloy taken, and therefore I nclude that the Mint Master must take to 120l, eight of Gold, 9l, of Alloy for, 20l, 1'5 : : 120. 9.

A Goldsmith having divers sorts of silver Bullion, z. 24l, weight of 7,oz, fine, 30l, weight of 9,oz, ne, and 16l, weight of 12,oz, fine, would melt all ese together, so as 1l, weight of the Masse must are 10, oz, fine, whether ought he to melt any Alloy therewith, and how much?

24 + 30 + 16 = 70.
× 7 × 9 × 12
168 + 120 + 192 = 630

Then if 70 630 : : 1.9. Therefore for answer 1l, weight will be 9,oz, fine. But by the Tenor of the uestion the Goldsmith should have 10,oz, fine, herefore this mixture is not fine enough by one

 ounce

ounce in every pound weight, so that no Alloy is to be mixed, but contrariwise more fine silver must be put unto it, which to finde worke by Alligation alternate is thus:

$$10 \left\{ \begin{matrix} 9 \\ \\ 12 \end{matrix} \right\} \left\{ \begin{matrix} 2 \\ \\ 1 \end{matrix} \right.$$

Signifying that for every 2l, of the three Bullions mixed together, there must be taken 1l, of fine silver: Lastly I conclude 2. 1 : : 70. 35, therefore the whole Mash 70l, will require 35l, of fine silver to be incorporated, to make 1l, weight 10,oz, fine.

For proofe I say.

24l, weight	— 7,oz, fine	=	168
30.	9,oz,	=	270
16.	12	=	192
35.	12	=	420
105	at 10, oz. fine	=	1050.

	70l, weight	at 9,oz,	=	630
Or	35	at 12,oz,	=	420
So	105	at 10,oz,	=	1050.

CHAP.

Chap. XIII.

Of divers usefull Propositions concerning the composition of Medicines.

1. WEE consider all Medicines either in their Quantities, Qualities or Prices.

2. The weights which Apothecaries use, are Pounds, Ounces, Drams, Scruples, and Graynes; in a pound there are 12 ounces, in an unce 8 Drams, in a dram 3 scruples, in a scruple 20 raynes: These alter from Troy weight something n their subdivisions; the Characters for Pound is ℔, for Ounces ℥, for Drams ʒ, for Scruples ℈, for Graynes gr.

3. You are to observe in the workes following, hat all the Quantities be brought to one denomination by reduction into the least tearme mamed, as lb, nto ℥ by multiplying by 12. ℥ into ʒ by multiplying by 8. ʒ into ℈ by multiplying by 3. ℈ into gr, by ultiplying by 20: or otherwise lb into ʒ by multiplying by 96, and lb into ℈ by multiplying by 288. r by putting them into Decimalls of an Ounce, for the

the greatest quantity commonly being ℥, pounds are soone reduced, for by the little Table annexed you may see that in 2l, there are 24℥, in 3l, 36 &c. Then by the Decimall Tables annexed, you may take out the Decimalls belonging to Drams, Scruples, and Graynes.

2	= 24 ℥
3	= 36
4	= 48
5	= 60
6	= 72
7	= 84
8	= 96
9	= 108
10	= 120 &c.

As the Decimall of 25 ℥, 4 ʒ, 1 ℈, 15 gr, is 25'57291 : But in the following Propositions there will be no great need of this decimall work, except in the values.

4. The qualities, faculties, and vertues of Medecines are considered in respect of us and not of themselves, for those simples are called temperate, that brings no change in our bodies in respect of heate, cold, moistures, and drynesse; those hot which have power of heate, those cold which worke cold, and so likewise some moist and some dry; so that all Medicines and simples are considered in quality in some of these wayes, either as they are hot or cold, moist or dry, or as they are temperate; which may likewise be taken two wayes; for either a Medicine may be said

ʒ	
7	875
6	75
5	625
4	5
3	375
2	25
1	125

℈	
2	08333
1	04167

15	03124
10	02083
5	01041
4	00831
3	00623
2	00416
1	00208

to

to be temperate in that it is neither hot or cold, and yet may be moist or dry, or in respect it is neither moist or dry, and yet may be hot or cold; or lastly and generally it may be said to be temperate in respect it is neither hot, cold, moist or dry.

5. The differences of these qualities are distinguished into degrees; for a simple or Medicine according to its expresse in quality, is accounted to proceed from the temperature towards any other quality in 4 degrees, and so a Medicine is said to be hot, cold, moist, or dry in the first, second, third, or fourth degree: As concerning the prizes, and the due proportioning thereof, it shall be spoken of in the Propositions following.

Prop. 1. To augment a Medicine in quantity, keeping the Proportion given.

Summe up the Quantities of the Medicine given; then say; as the summe of the Medicine given, is to the augmentation, so is the quantity of each parcell of the Medicine given, to the quantity of the augmented Medicine desired; Examp.

Unguentum

the greateſt quantity commonly being ℥, pounds are ſoone reduced, for by the little Table annexed you may ſee that in 2l, there are 24 ℥, in 3l, 36 &c. Then by the Decimall Tables annexed, you may take out the Decimalls belonging to Drams, Scruples, and Graynes.

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8 =	96
9 =	108
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As the Decimall of 25 ℥, 4 ʒ, 1 ℈, 15 gr, is 25'57291: But in the following Propoſitions there will be no great need of this decimall work, except in the values.

4. The qualities, faculties, and vertues of Medecines are conſidered in reſpect of us and not of themſelves, for thoſe ſimples are called temperate, that brings no change in our bodies in reſpect of heate, cold, moiſtures, and dryneſſe; thoſe hot which have power of heate, thoſe cold which worke cold, and ſo likewiſe ſome moiſt and ſome dry; ſo that all Medicines and ſimples are conſidered in quality in ſome of theſe wayes, either as they are hot or cold, moiſt or dry, or as they are temperate, which may likewiſe be taken two wayes; for either a Medicine may be ſaid

ʒ	
7	875
6	75
5	625
4	5
3	375
2	29
1	125
℈	
2	08333
1	04167
15	03124
10	02083
5	01041
4	00831
3	00623
2	00416
1	00208

to

to be temperate in that it is neither hot or cold, and yet may be moist or dry, or in respect it is neither moist or dry, and yet may be hot or cold; or lastly and generally it may be said to be temperate in respect it is neither hot, cold, moist or dry.

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Prop. 1. To augment a Medicine in quantity, keeping the Proportion given.

Summe up the Quantities of the Medicine given; then say; as the summe of the Medicine given, is to the augmentation, so is the quantity of each parcell of the Medicine given, to the quantity of the augmented Medicine desired; Examp.

Unguentum

Unguentum album Campharatum.

℞.	*Oyle of Roses* —— ℥	12	*or*	96 ℥	
	White Wax ————	3	*or*	24	
	Ceruse —————	6	*or*	48	
	Camphyre bet with oyle of Roses —	0	*or*	02	
				170	

I desire to augement this, or make up an oyntment consisting of 210 ℥.

Therefore say,

```
170. 210 : : 96. 118 : : 24. 29½ : : 48. 59¼ : : 2. 2½ -
          96            210           210
         ----           ----          ----
         126             24            48           420(2½ -
        189              48            96           340
        -----           ----          ----          ----
170)20161 (118         5040 (29      10080 (59¼      80
      31                164            85
      17                153            158
      ---               ----           153
      146               110            ----
                                        50
```

Answer, you must take of Oyle 118 ℥, of white Wax 29½ ℥, of Ceruse 59¼ ℥, of Camphyre 2½ ℥, to make up the Medicine 210 ℥.

Prop.

Prop. 2. To diminish a Medicine in quantity, keeping still the proportion of the given quantities.

This is performed as the last Proposition; for as [t]he sum of the Quantities of the given Dose, is to the [s]umme of the Quantities of diminution, so is the [s]everall Quantities of the Dose given, to the seve[r]all Quantities of diminution. The worke as in the [f]ormer.

[P]rop. 3. To finde out what Quantity of any Ingredient or simple is contained in any Quantity of a Composition.

Consider the Composition, and summe up the In[g]redients: then say, as the summe of the great Com[p]osition is to the Quantity of your Dose; so is the [Q]uantity of the Ingredient proposed in the Compo[si]tion, to the quantity of that Ingredient in the Dose [p]roposed: Example, I have 1 ℥ of *Pilulæ sine qui[b]us*, I would know what quantity of Scamony is [c]ontained therein? For answer, I consider the Com[p]osition, and finde that in the whole there is contai[n]ed 3283: Therefore I say 3283, *96* :: *6*. 1 $\frac{1}{3}$; [a]nd I conclude that there being *6* Drams contained [i]n the whole Dose, there must be 1 $\frac{1}{4}$ ʒ in 1 ℥ of that [C]omposition.

Prop. 4. *To know the exact temperament and quality of any Medicine whatsoever.*

Hitherto have wee spoken of the Quantities, now come wee to the Qualities, and for finding the Quality emergent:

1. Dispose the quantities of all your simples into one row, setting orderly by them their severall quantities, and if any of the Ingredients be compounded, you must first learne its quality.

2. Multiply each quality by its owne quantity, and substract the hot from cold, moist from dry, or contrarily, and set downe the difference of these Products: For as the summe of all the quantities, is to the difference of the Products, so is unity or one, to the quality emergent, which is alwayes of the same kinde that the greater Product was of: Example, I have 4 ℥ of a simple cold in 2°, moist in 1°, 9 ℥ hot in 3° and temperate. 3 ℥ hot in 2°, and dry in 2°, 6 ℥ hot in 1°, moist in 4°. 4 ℥ cold in 3°, and Moist in 2°. I would know if these were compounded, what should be the temper resulting? For answer, I order them as in the Example.

Hot

	℥	°		
Hot.	5 —	3	15	
	3 —	2	6	
	6 —	1	6	27
Cold.	4 —	2	8	
	4 —	3	12	20
	22			

$$\frac{20}{7}\quad\frac{7}{22} = \frac{10}{1}\ \text{Heat.}$$

	℥	°		
Moist.	4 —	1	4	
	5 —	0	0	
	6 —	4	24	36
	4 —	2	8	
Dry.	3 —	2	6	6
	5 —	0	0	

Moist.

$$\frac{30}{27}\ (1\tfrac{3}{27}$$

I first set downe my quantities of heate and cold, and multiply them by their qualities: I adde up both the Products, & substract the greater from the lesser, and divide the Remainer by the quantity of ounces, the Quotient gives me $\frac{1}{3}$ o of heate, and 1 $\frac{3}{27}$ of moist, because the Products 27 of dry, and 36 of moist, were the biggest; and now therefore I conclude that the resultment of this Medicine in quality was $\frac{1}{3}$ deg. of heate, and 1 $\frac{3}{27}$ of moist.

Ex.

Ex. 2. Where the quantities are the same.

What is the quantity emergent of 1 ℥ hot in 3°, moist in 2°, mixed with 1 ℥ cold in 1°, and dry in 3°? Answer, in 1° of heate, and ½ of drought.

Hot.	1 — 3	3		
Cold.	1 — 1	1		
	2)	2 (	1 Heate.	

Moist.	1 — 2	2	
Dry.	1 — 3	3	
	2	1	Dry.
		$\frac{1}{2}$	

Ex. 3.

	3				
	31 — 3	93			
Hot.	11 — 3	33	136		
	5 — 2	10			
	2 — 0	0		$\frac{57}{83}$ Heate.	
	17 — 2	34			
Cold.	15 — 3	45	79		
	2 — 0	0			
	83				

	11 — 4	44		
Dry.	2 — 3	6	93	
	7 — 1	7	19	
	9 — 4	36	74	($1\frac{31}{43}$ Drought.
Moist.	5 — 2	10	43	
	9 — 1	9	31	
	43			

Examp. 4. Pilulæ Synoglossi.

	ʒ	call.	fri.	hum.	sic.	tem.
Radicum synoglossi sic.	48	0	2	0	2	0
Sem. Hyoscijami albi	48	0	3	0	1	0
Opii præparat.	48	0	4	0	0	0
Myrrhæ.	6	2	0	0	2	0
Thuris Masc.	5	2	0	0	1	0
Croci.	2	2	0	0	1	0
Castorei.	2	2	0	0	2	0
Styracis Calamitæ.	2	1	0	0	0	0

161 The summe of the Quantities in Drams.

The worke.

	q.	c.	f.	h.	sic.
1	48	0	96	0	96
2	48	0	144	0	48
3	48	0	192	0	0
4	6	12	0	0	12
5	5	10	0	0	5
6	2	10	0	0	2
7	2	10	0	0	4
8	2	2	0	0	0
	161	44	432	0	167

432 — 44 = 388
167 — 9 = 167

$161)\ 388\ (2\frac{66}{161}$ *frig.*
$\frac{322}{66}$

$161)\ 167\ (1\frac{6}{161}$ *sic.*

Wherefore I conclude the said Pills to be cold in 2 degrees and almost a halfe, and dry in one degree and something more, the same may be pronounced of any other Dose.

Prop. 5. *To augment in Quality a Medicine to any degree proposed.*

It is requisite for an Apothecary to know the due resultment of any Medicine, as *Zenertus* affirmeth *Lib.* 5. *Instit.* *Pars* 3. *Chap.* 1. which is taught in the last; and in the same Chapter, amongst many o-ther observations, he saith, *Interdum vis Medica-menti est debilis, quam validioris admistione intendere, contra nonnunquam vehementior, quam debilioris ad-ditione remittere oportet.*

The Rule.

1. Dispose all the quantities given either of simples or compounds in their exact degrees which are to be taken and tempered out of Medicines contrary betwixt themselves, *viz.* hot with cold, and m

moist with dry, otherwise no exact temperament can be chosen.

2. As in the last Prop multiply the quantities and qualities, and substract the Products, as was there taught.

3. Then alwayes chuse one of the simples a degree higher then the propounded degree, as if I would raise it from 1° of heat to 2°: Then I chose among the simple Medicines one in 3° degrees of heate, or if I finde none, then I must chuse a simple of that degree, this done by the degree you have given, multiply the summe of the quantities, and from the Product substract the difference of the Product of either Temper; the Remainder divide by the difference of the degree desired, and the degree from which it is desired to be augmented.

Examp. 1.

I have of simples; 1 ʒ dry in 4°, 2 ʒ dry in 3°, 1 ʒ dry in 1° and 2 ʒ dry in 2° this being mixed with other simples, *viz.* 1 ʒ cold in 2°, and 1 ʒ cold in 1°, I finding the temper of this mixture to be onely $1\frac{1}{2}$ degrees of heate, desire that it may be augmented to two degrees.

```
        3.       0.
     1 — 4   4
H.   2 — 3   6
     1 — 1   1   15
     2 — 2   4        12
     ----------
C.   1 — 2   2   3
     1 — 1   1
     --------
     8
     2   1) 4 (4.
     --
     16
     12
     --
      4
```

I finde that by adding 43 to the 23, that is hot in 3°, (because it is one degree higher then the degree desired) to augment the heate to two degrees.

The proofe.

```
                  3.      0.
 2            1 — 4    4
 4  —     6 — 3   18   27
 -                       3
 6   H.   1 — 1    1    --
          2 — 2    4
          -----   --
          1 — 2    2   3)  24  (2.
          1 — 1    1       12
          ------
         12
```

Examp

Examp. 2. *Where the heate is raised 2 degrees.*

```
  3 — 3 | 9  |                3 - 3   9 |
 15 — 3 |45  |               15 - 3  45 |
  7 — 4 |28 90   7           66 - 4 264
  8 — 1 |  8|41  8            8 - 1   8 326
 11 — 2 |22|49               11 - 2  22  41
  5 — 1 | 5|56                5 - 1   5 285
  7 — 2 |14|                  7 - 2  14
 56                         115
  3
168
 49
119 (59
2)
```

Prop. 7. *To diminish a Medicine in quality from any degree whatsoever.*

This is but the converse of the former, for taking one degree of cold, or that way from the degree of heate given more then the degree proposed, and working as before in the last Proposition you have the quantities to be added to the quantities of the degree taken.

Prop. 7. To reduce any Medicine proposed to any degree of quality whatsoever.

1. This is wrought by the Rule of Alligation, and therefore you must have your qualities to ascend from 1 to 10, that a difference amongst them may be found out, accounting the temperate equall to 5, and then adding degrees of hot and dry to 5, and substracting cold and moist from 5,

Hot	9	8	7	6	5	4	3	2	1.	Cold
and dry.	4	3	2	1	0	1	2	3	5.	and moist.

as in these figures, where the upper figures may be tearmed the differences, the lower being the quantities the Cypher standing for temperate and the degrees of hot and dry being accounted towards the left hand, cold and moist towards the left.

2. In this Rule the operation is still wrought by the differences or higher Numbers, and not with the degrees themselves; as if you would set downe 3° of heate, and 1° of cold, you must use the numbers above, *viz.* 8 and 4.

3. Therefore when a question in this way is proposed, set downe the differences in their order, from the highest degree of heate downwards, or contrarily, and then by the Rule of Alligation alternate, take the differences from the degree proposed alternate, and subscribe the Doses or Quantities, as in the Example following.

Example

Example 1. I have simples some hot in 3°, in 2°, some temperate, some cold in 2°, and in 4°; It is desired that a Medicine may hereof be compounded hot in the 1°, what Doses or Quantities must be taken of each?

Answer, for to make the Medicine 14 ℥, you must take 8 ℥, of the first, or of hot quantity in 3°, of the second and third, of either 1 ℥, and of the two last of either two; or changing the Alligation, you may alter the quantities as in the second Alligation.

			diff.	quan.	q.
Hot 1°	3° Hot.		8	5+3	8
	2° Hot.	*In their*	7	1	1
	0° Temp.	*differen-*	6 { 5	1	1
	2° Cold.	*ces to be*	3	2	2
	4° Cold.	*set thus.*	1	2	2
					14.

The proofe.

diff.	*Or thus.*	q.	d.		q.	d.	
8	5+1	6 – 8	48		8 – 8	64	
7	3	3 – 7	21		1 – 7	7	
6 { 5	2	2 – 5	10		1 – 5	5	
3	1	1 – 3	3		2 – 3	6	
1	2	2 – 1	2		2 – 1	2	
		14)	84	(6	14)	84	(6
			84			14	
						84	

The

The proofe of this Rule ariseth out of Prop. 4. For if you multiply the quantities by the differences, and divide the product by the summe of the quantities, and if the Quotient be equall to the quality proposed, then you have wrought well.

Examp. 2. And heere are all the severall cases of Alligation to be applyed, as in this Example where the quallities of all, but the quantity of one simple is expressed, and it is desired what quantities of the rest must be taken, to make up a Medicine of a quality given. As for Example, I have 4 ℥, of a simple hot in 3° to be mixed with qualities hot of 2° and 1°, and with qualities cold of 2° and 4°, what quantities of each of the later must I take to make the Medicine onely in 1° of heate? Answer;

Then if 5. 3 :: 4. 2 ℥ $\frac{2}{5}$
5. 4 :: 4. 3 $\frac{1}{5}$
5. 1 :: 4. $\frac{4}{5}$
5. 2 :: 4. 1 $\frac{3}{5}$

The severall ounces and parts to be taken of the respective qualities to be mixed with the 4 ℥ of 3° in heate, and yet the quality to be 1° in heate.

Another

Another variety may be in such a question as this: I have simples of severall qualities, what quantity must I take of each to make a Dose of 14 ℥, and yet the quality to be some meane amongst the qualities of the simples given? But because these and the like have beene largely handled in Alligation alternate I omit the resolution.

And heere I might againe observe to the Reader how by these differences of qualities, the former Propositions may as easily be wrought; as in the proofe of the 1 Examp. Prop. 5. I would know the exact temperament of that Medicine.

— 4° H.	In their Differences thus.	1 - 9	9
— 3° H.		6 - 8	48
— 1° H.		1 - 6	6
— 2° H.		2 - 7	4
— 2 Co.		1 - 3	3
— 1 Co.		1 - 4	4
		12)	84 (7
			84

I finde the difference of the quantity to be 7, which signifieth 2° of heate, answerable to the proofe of that Example in Prop. 5. I will now passe to the prizes of Medicines.

Prop.

Prop. 8. To finde out the value or price of any quantity of a Medicine, having the values of the simples first given.

This is found out by Alligation Mediall, for as the summe of the Quantities, is to the summe of the severall products of the quantities in their prices, so is the quantity proposed to his price.

As for Example, if an Apothecary compound the oyntment *Unguentũ Pectorale, viz.* New Butter 6℥ at ob. the ℥, 4℥ of the Oyle of sweet Almonds at 1d. ob. the ℥, 3℥ of the Oyle of Chamomel at 3d. the ℥, of Violets at 4d. the ℥, 2℥ of the fat of a Ducke at 3d. the ℥, of a Hen at 2d. the ℥, 1℥ of Flower de Luce Roots at 2d. the ℥, and 3℥ of white Bee Wax at one penny the ounce, how may he sell 4 ounces of this Medicine? Answer, 7d. ob.

$$\left.\begin{array}{l} 6+4+3+3+2+2+1+3=24 \\ \times 1\times 3\times 6\times 8\times 6\times 4\times 4\times 2 \\ \hline 6+12+18+24+12+6+4+6=90 \end{array}\right\}\begin{array}{l} 24.\ 90::4.\ 15 \\ \\ \&\ 15.\text{ob}=7\text{d-ob}. \end{array}$$

An Apothecary useth in Powder, 6l of Sugar of 3s, the l. two pound of Lycoras at 6d, the l, three pound of Anniseeds at 10d, the pound, and one pound of Fennell-seede at 6d, the pound; what is 3l, of this mixture worth? Answer 5s, -6d.

6+

$$\left.\begin{array}{l} 6+2+3+1=12. \\ \times 36 \times 6 \times 10 \times 6 \\ \hline 216+12+30+6=264 \end{array}\right\} \begin{array}{l} \text{And } 12.\ 264 :: 3.\ 66. \\ \text{or } 5s, — 6d. \end{array}$$

Thus have I delivered 8 usefull Propositions concerning Medicines, never to my knowledge written on before.

CHAP. XIV.

The Rule of Position.

1. THis is called the Rule of Position, because any number is taken for to worke the question by, and placed instead of the number sought, after the position wherefore you examine the question according to the Tenour thereof, and if your Position fal true then you have satisfied the Question, if false (from whence it is called the Rule of false Position) then the true number is found out by this Rule, whereof there are two kindes: 1. The single Rule. 2. The double Rule.

2. The single Rule is to be used when there is some partition of numbers in parts proportionall; and

and for finding out the truth, place the number found by the argumentation first in the Golden Rule, in the second place put downe the *Hypotheticall* or Number supposed, and in the third place the given number, the Numbers being thus disposed, the fourth Proportionall shall be the Number sought.

Examp. 1. Three men, *viz.* *A. B. C.* buy a Mannor, costing 2700l, and *B* is to pay double, what *A* must pay, and *C* triple to that which *B* payeth; how much ought every of them to pay? For answer I suppose A payd 6l, therefore *B* payd 12l, and *C* payd 36l, but 6+12+36 maketh 54l. which by the intent of the question ought to have beene 2700l, wherefore by this Position I want of the truth; yet according to my Rule, if 54l. come of 6l : : 2700l. 300. Therefore I conclude *A* payd 300l. *B* 600, and *C* 1800, for.

$$\left.\begin{array}{r}300\\+600\\+1800\end{array}\right\} = 2700\text{l. The Total.}$$

Examp. 2. One bought thirty yards of Taffaty, and 40 yards of Satten, which cost him 350s. but every yard of Satten cost double so must as a yard of Taffaty, what did the yard of Taffaty cost? For answer I suppose a yard of Taffaty cost 4s. therefore a yard of Satten cost 8s. therefore 30 yards of Taffaty cost 120s. and 40 yards of Satten cost 320s. but 120 +320

+ 3 0 = 440s. which should be but 330s. I therefore say, if 440. 4 : : 330. 3 The answer is 3s. and so much a yard of Taffaty cost, and a yard of Satten 6. For proofe 30 yards of Taffaty at 3s. cost 90s. and 40 yards of Satten at 6s. cost 240, and 90+240 = 330.

If the question have a Fraction in it, it is best for most facility in proceeding to chuse such a number for the position as hath the parts exprest in the question as in the example following.

Examp. 3. One sayd he knew not what money he had in his purse, but he knew that the third part, and the fourth part, and the fifth part thereof would make just 94l. what money had he? For answer I chose 60 for my position which hath the parts exprest (and such choyse is easily made by multiplying the parts together) therefore I suppose he had 60l. But the third part thereof (20) the fourth part thereof (15) and the fifth part thereof (12) that is to say 20+15+12 = 47, and by the question it should be 94l but now I say if 47. 60 : : 94. 120, wherefore I conclude he had 120l. in his purse; and for proofe I say the third part thereof is 40, the fourth part 30, and the fift part 24, now 40+30+24 = 94 according to the intent of the question.

In questions wherein a number constant, or permanent is given; substract it for a time from the given summe, and after operation restore it againe, because it doth not rise and fall proportionably as the Fractions doe.

Ex. 4. One ſaid if I had in my ſtock as many moe as I have, together with the halfe, the third and fourth parts of theſe I have, and one *overplus* I ſhould juſtly have 630. How many had he? For anſwer if I finde a number which being twice taken or doubled and the halfe, third and fourth parts thereof will be 629, I ſatisfie the queſtion, for then adding to it, the permanent Number 1 it makes up juſt 630. ſuppoſe my Number was 12 (becauſe it hath the parts required) but then 12+12+6+4+3 make up but 37, and ſhould be 629. Therefore I ſay if 37 come of my Poſition 12, of what commeth 629: The Rule anſwereth 204, and for proofe 204+204+102 +68+51 will be juſt 629, and now reſtoring the 1 before ſubſtracted, 629+1 will be 630 according to the intent of the Queſtion.

37. 12 :: 629. 204.

12

7548 (204

148

Examp. 5. One had ſpent the $\frac{2}{3}$ and the $\frac{1}{5}$ of his ſtock, and had onely 36 ſhillings remayning, what was his ſtock? For anſwer ſeeke ſuch a Number as having the $\frac{2}{3}$ and the $\frac{1}{5}$ thereof abated leaveth remaining 36. Suppoſe therefore the ſtock was 15, whoſe $\frac{2}{3}$ (10) and $\frac{1}{5}$ (3) that is 10+3 being taken from 15 reſteth 2 remaining, which by the intent of the queſtion

queſtion ſhould be 36, therefore by the Rule if 2. 15 :: 36. 270, which was the ſtock, for the two thirds thereof 180, and the fifth part 54, added maketh 234, and 270 -- 234 = 36.

Note that when the Fraction or parts expreſſed in any queſtion to be ſubſtracted exceed an unite, ſo that in reaſon it cannot be ſubſtracted from the Poſition or number put, that then it is a queſtion abſurd, and utterly impoſſible, and therefore had the queſtion before required been put, that $\frac{2}{3}$ and $\frac{3}{5}$ thereof ſhould be abated, as it was but $\frac{2}{3}$ and $\frac{1}{5}$ it had beene impoſſible, for $\frac{2}{3}$ of 15, in the poſition is 10 and $\frac{3}{5}$ is 9, now 10+9 = 19, and how is it poſſible to ſubſtract 19 from 15.

Examp. 6. What Numbers are they whoſe $\frac{2}{3}$ of of the one is $\frac{3}{4}$ of the other: for anſwer, I take a number that hath ſuch parts as 12. whoſe $\frac{2}{3}$ is 8, then I ſeek to know what number 8 is $\frac{3}{4}$ and put ſuch a number for my poſition as hath 4 for the denominator, as 20 whoſe $\frac{3}{4}$ is 15. but ſhould be but 8. and now if 15 come of my Poſition 20 of what commeth 8? Anſw. 10 $\frac{2}{3}$ of 12. Therefore 1 and 10 $\frac{2}{3}$ are the two numbers required; for as 8 is $\frac{2}{3}$ of 12, ſo is 8 likewiſe $\frac{3}{4}$ of 10 $\frac{2}{3}$.

Examp. 7. What two Numbers are they, whereof $\frac{1}{2}+\frac{1}{3}$ of the one is equall to $\frac{1}{4}+\frac{1}{5}$ of the other? for anſwer, take ſuch a number as hath parts like the Fractions as 24, whoſe $\frac{1}{2}$ 12 + $\frac{1}{3}$ 8, is 20. Then I put 40 for the ſecond Number whoſe $\frac{1}{4}$ 10 + $\frac{1}{5}$ 8, is 18. which

which according to the queſtion ought to have been 20. But now I ſay if 18. 24 :: 20. $44\frac{4}{9}$. wherefore I conclude 24 and $44\frac{2}{3}$ are the two numbers deſired, for the $\frac{1}{2}$ more the $\frac{1}{3}$ of 24 is 20, and ſo much is the $\frac{1}{4}$ + $\frac{1}{3}$ of $44\frac{4}{9}$.

In the two laſt Examples there is not two poſitions made, for both 12 and 24, the Numbers I take in each queſtion, continue and allwayes remaine one of the true numbers, that ſerve to anſwer the queſtion without any alteration; but neither 12 or 24 (which I take for my Poſition) are any of the true.

4. But if there be no partition in numbers to make a proportion, then muſt you uſe the *Rule of double Poſition*; that is, you muſt twice make a Suppoſition, and if at either time you hit upon ſuch a number as will ſatisfie the queſtion, you have performed it, if not, obſerve the errors, and whether they were greater or leſſer then the reſolution required, and according marke them with the ſignes of + or −.

5. After you have ſet downe the errors, multiply them by the contrary Poſitions, and if the Errours were both too great or both too little, ſubſtract both the one Product from the other, and the one errour from the other, and divide the difference of the Products by the difference of the Errours; but if the errours be of divers kindes adde the Products, and likewiſe the difference together, and divide the ſumme of the Products by the ſumme of the Errours: The

Quotient

Quotient gives the Number sought, for the proportion of the Errours is the same with the proportion of the Excesses or defects of the suppositious numbers, from the numbers sought.

Examp. *A*. *B*. and *C*. doe agree to divide 100s. amongst them thus, *B* shall have more shillings then *A* by 3, and *C* shall have 4s. more then *B*, how many shillings must each have?

For answer,

I suppose *A* shall have 33, then *B* 36, and *C* 40 But the totall of 33+36+40 = 109 and should have beene but 100, therefore

The 1st. position 33.	the 2d. po. 31.	
Errour + 9.	Errour + 3	
Crosse Multip. 99.	Prod. − 279	
	99	
	Diff. Pro. 180	(30.

	9	
	3	6)
Differ.	6	

I erre 9 more then I should; I make my second supposition to be 31 for *A*, therefore *B* 34, and *C* 38. But 31+34+38, are 103, and should but be 100, therefore I erre againe 3 too much; therefore multiplying crossewise, and because the errours were both too great, I abate both the errours and products, and according to the Rule divide 180 the *Difference* of the Products by 6 the difference of the errours, and finde that *A* had 30s, therefore *B* 33, and *C* 37, and 30+33+37 = 100.

And this is the resolution where both the errors are too great.

Now suppose in the same question *A* had 25, therefore *B* 28, and *C* 32; but 25+28+32 = 85. The Error too little by 15. Againe if I suppose *A* 27, then *B* 30, and *C* 34; But 27+30+34 = 91, too little by 9: And because the errours are alike, I worke as before, and finde the answer for *A* to have 30s. as before.

The 1 *Po.*	25	*The* 2 *Po.*	27
Errours	− 15	*Errours*	— 9
Crosse Pro.	225	——	405
Differ. of Prod.	180		
Differ. of Er. 6)	180	(30	

Againe suppose in the same question I had, suppose *A* had 34, therefore *B* 37, and *C* 41. Now 34+37+41 = 112, which ought to be but 100, therefore too much by 12; In the second Position let *A* be 20, therefore *B* 23, and *C* 27. But 20+23+27 = 70, too little by 30; and now seeing the errours are unlike, after alterne multiplycation the Products and Errours are summed up, and according to the Rule the Quotient is found 30 for *A* as before, and so have you this example in every variety.

The 1 *Po.*	34.	*The* 2 *Po.*	20
The Er.	+12	*Err.* —	30
Crosse Prod.	1020	—	240
Summe of the Prod.	1260 (30		
Sum of errors 42)	126		

Examp. 2. Two men *viz. A* and *B* snatched at 200s. and after agreement *A* said to *B* give me the $\frac{1}{3}$ of the shillings you snatched, and I will give you the $\frac{1}{5}$ of mine; this done they had either of them

50s.

50s. How many therefore had each snatched?

For answer, I suppose *A* snatched 30. Now $30 - 10 + \frac{1}{5}$ of 70 *B* had that is 14, makes *B* to have 56+10, and *A* to have but

The 1 *Po.* 30.	*The* 2 *Po.* 45		
Err. — 16.	*Err.* — 9		
The Crosse Pro. 270 — 720			
The Diff. of Pro. —	450		
The Diff. of Err. —	42	$(64\frac{2}{7}$	
	30		

$20+14 = 34$, which should be 50, and so 16 too little; I therefore now suppose *A* snatched 45; Now $45 - 15 + \frac{1}{5}$ of 55 which *B* had that is 11 make *B* to have 44+15, and *A* to have 30+11, that is 41, and should have 50, so the Errour is 9 too little.

And now according to the Rule I finde *A* snatched $64\frac{2}{7}$, and *B* snatched $35\frac{5}{7}$.

And for proofe.

A having $64\frac{2}{7} - 21\frac{3}{7}$ and $\frac{1}{5}$ of $35\frac{5}{7}$ which *B* had that is $7\frac{1}{7}$, *A* had $42\frac{6}{7} + 7\frac{1}{7} = 50$ }
B had $28\frac{4}{7} + 21\frac{3}{7} = 50$ } According to the Condition of the Question.

Examp. 3. *A* having stolne some number of shillings was stayed by *B*, to whom he gave the $\frac{1}{2}$ of his shillings, but he returned 12s againe, after he was stayed by *C*, to whom he gave $\frac{1}{2}$ of his shillings, and he returned back 7s. after he was stayed by *D*, to

whom he gave $\frac{1}{2}$ of his shillings, and he returned 4s. And after all this he had 20s. how many shillings did he steale at first? For answer, I suppose *A* had stolne 40.

Now 40 −	*The 1 Po.* 40 *The 2 Po.* 60	
20 + 12 =	*The Err.* 4'5 *Err.* — 2	
32 & 22 −	*The Crosse Prod.* 80 -- 370	
16 + 7 =	*The Diff. of Prod.* -- 190	(76
23 & 23 −	*The Diff. of Er.* 2'5) 175	
$11\frac{1}{2}$ + 4 =	15'0	
$15\frac{1}{2}$ but should have	15'0	

been 20, therefore the errour is too little by $4\frac{1}{2}$ or 4'5. I suppose therefore now *A* had stolne 60, now 60 − 30 + 12 = 42 and 42 − 21 + 7 = 28 and 28 − 14 + 4 = 18, but should have beene 20, therefore 2 too little, and so now I finde by the worke that *A* stole at the first 76 shillings.

And for proofe I say, $\left\{\begin{array}{l} 76-38+12=50 \\ 50-25+7=32 \\ 32-16+4=20 \end{array}\right.$ The true intent of the Question.

And for conclusion of this Rule, any question whatsoever if not impossible may thereby be resolved; if by comparing, adding, substracting, or Proportion, you could prove that question if the true resolution was given, for otherwise the question cannot

not be resolved, because you cannot come to know what the errours were at the Positions; but because all such questions at one worke is taught in the second part by a most patheticall way of Art, I shall refer you thither, and will now procced to shew you the use of the Tables of Interest annexed to the end of this part.

CHAP. XV.

Of Compound Interest.

AT the end of this booke there are five Tables for resolving all questions concerning Interest of money and Annuities, all made after the proportion 100. 108. or at the rate of 8l. *per centum.*

Now wee consider these Tables in their construction, explication, and use, and first of their construction.

1. The increase of use upon use is after a Geometricall Proportion continued, therefore the 1 Table is nothing but the continuing the tearmes 100. 108 = in a Geometricall Progression for each yeere; as 108. for the first yeere 116'64, for the second *&c.* But to doe it for halfe yeeres and quarters, you must finde a meane proportionall betwixt 100. 108. by ta-

 king

king the square roote of 10800. which is 103'923, and betwixt 1000 and 103'923, which is 101'942, &c. now having the two first tearmes 100. 101'942, &c. you are to continue it out for quarter of yeeres, and so is the first Table made, in imitation whereof if you please you may make Tables for 6 *per centum*, 5. &c. The second Table is a continuall proportion decreasing: The third Table increasing, and so the fourth: The fifth Table decreasing, and thus much for the *Construction*, now to the *Explanation*.

2. The first Table gives the Interest of 100l. after 8l. *per centum*, for every quarter of a yeere to 33 yeeres, and then for whole yeeres to 100: As for example, the use of 100l. for halfe a yeere is 103'923, which by the first Table of the decimall of pounds (to be used in this worke) is 103l. – 18s. – 6d. almost, and not 104l. as is usually taken; the same Table gives you the use of 10l. if you cut off one figure before the comma with your seperating line, and if 1l. of two, &c. so it will of 1000l. if you removing the comma one figure towards the right hand, and this is to be observed as well in this as in the rest of the Tables, that if the *Principall* whose *Interest* you seeke, be under 10l. you remove the comma, or *seperating* lyne two figures towards the left hand, if under 100l. and above 10l. then but one figure. The second Tables sheweth what 1. 10. 100l. due at the end of any yeere or quarter is worth in ready money; The uses of the rest of the Tables appeares by their superscription

ſuperſcription above them, and by Propoſitions following.

The uſe of the firſt Table. Prop. 1. To know what the Intereſt of any ſumme of money put out for any Number of yeeres or quarters, is worth at the end thereof.

Obſerving the Cautions in the laſt ſection, accorcing to your ſumme given, whether it be under 10l. 100l. or 1000l. enter with the yeares or quarters on the left hand, and having found the ſumme anſwering, propoſe the Golden Rule thus, If 1l. 10l. 100l. give in the yeeres propoſed the ſumme in the Table, then what ſhal the ſumme propoſed give? The which Analogy is to be obſerved in all the Tables and Queſtions following.

Queſt. 1. If 8l – 15s – 6. be put out for 2 yeeres 3 quarters, what is it worth at compound Intereſt at the end thereof? Anſwer, 10'843, or 10l – 16s – 10d.

Queſt. 2. If 75 – 11s – 2d. be put out for 32 yeeres and a quarter, what is it worth at the end thereof? Anſwer, 904l – 1s – 3d.

Queſt. 3. If 842l – 6d. be put out 3 yeeres at compound Intereſt, what is it worth? Or what is the Principall and gaine thereof? 1060l. 14s – 2d.

```
      1. 1'2357 :: 8'775.
              8'775
 1. Op.) 6.....
 1. Ex.) 86....          (10'843
         865...
       9886..
    10'843

      10. 11.9'6509 :: 75'5583
               7.5'5583
  Op.   36.......
 2. Ex. 957.....
        5983....        (904'0614
       59825...
      598250..
     8375563.
     904'0614

      100. 12.5'9712 :: 842'025
               8.42'025
   Op.   63......
 3. Ex.  252......
         25194....
        503894...
      10077696
     1060'7089
```

The

The use of the second Table. Prop. 2. To know what any summe of money due at the end of any Number of yeeres or quarters, is worth in ready money.

Take the number against the yeares and parts propounded and multiply it by the summe propounded, and divide it by 1l – 10l – 100l. *&c.* according to the cautions aforegoing, and the Quotient is the true answer.

What is the worth of 5l – 11s – 6d. due at the end of two yeeres 3 quarters in ready money? Answer, 4l — 10s.

What is the worth of 81l. 2s. due at the end of 9 yeeres and ½ worth in ready money? Answer, 39l. – 9d.

What is the worth of a lease that will give 750l, . 3d. at the end of 7 yeeres in ready money? An.

Oper.) 1.'8.092 :: 5'575 Op.) 10. 4'8136 :: 81'1
1 Ex.) 5'575 2 Ex.) 81'1

40.		48136	
566.	(4'5012	48136	(39'0383
4046.		385088	
40460.		39'0383	
4'5012			

Op.

Op. 3. Ex. 100. 58'85 :: 750'21.25.

```
        750'21.25
          .58'85
        --------
          37511.
         50017.
        500170.
       3751062      (430'5002
       --------
       430'5002
```

Of these two last Propositions and so likewise of the following ; the question might be put on the principall by Division , by demanding the question thus, 255l. came of a summe in 7 yeeres, what was the principle for the first Prop ?

Or thus in the second Prop. 39l – 9d. is the present worth of a certaine summe payable at the end of 9 yeeres, and ½, what was that summe ? but the practice of these, lest I should be somewhat tedious, I leave to the ingenious Reader.

The use of the third Table. Prop. 3. To know what any Annuity payable yeerely, halfe yeerely or quarterly, and forborne any time under 50 yeeres, is worth or doth amount to at the end thereof.

Multiply and worke as before observing the cautions belonging to the question ; but heere you must observe

observe that this third Table hath three parts, you must worke by the first if the payments be yeerely, by the second if the payments be $\frac{1}{2}$ yeerely, and by the last if quarterly.

Ex. 1. 5l -- 4s - 6d. being an Annuity *per annum*, and forborne 5 yeeres, what ought to be payd at the end thereof accounting Interest upon Interest at 8 *per centum*? Answer, 30l - 13s. 1d —.

If the Annuity had beene quarterly to have beene payd, and forborn 5 yeeres, then the answer had been 26l - 4s - 7d.

Ex. 2. One ought to have payd each halfe yeere to another 55l. but did discontinue 9 yeares and an halfe, what ought he to pay? Answer, 1400l — 11 — 6d.

Examp. 3. A quarterly Annuity of 146l - 10s. being forborne 11 yeeres and three quarters, it is desired to know the worth of it? Answer, 11087l. 3 s. 7d.

1. 5'8.667 : : 5'225.
5'225

Op. 1 Ex.) 293.
1173. (30'6534
11733.
293335.
30'6534

1. 24'1:591 :: 5'225. 126'2311.

```
        5'225
   1203.
   04831.        (126'2311.
  048318.
 1207955.
 --------
 126'2311.
```

Op. 2. Ex.) 10. 254'65 :: 55.

```
          55
       ------
       127325
       127325   (1400'575
       ---------
       1400'5.75
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Op. 3. Ex.) 100. 7567'906 :: 146'5

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         146'5
       ---------
        37839.
       454074.
      3027362.
      7567906.
     -----------
    11087'181.
```

The use of the fourth Table, Prop. 4. to know what any annuity payable yearly, halfe yearly, or quarterly, is worth in ready money.

The work is as the last, the examples follow.

Ex. 1. There is an annuity 6 l. 10 s: 4 d. *per annum*, to be paid for 12 yeares, what is it worth in ready money? *Ans.* 49 l. 2 s. 2 d.

If it should have been quarterly, the answer would have been, 202 l. 4 s. 9 d.

Ex. 2. There is an annuity of 52 l. 10 s. to be paid halfe-yearly for 4. years and a halfe, what is it worth in ready money? *Ans.* 391 l.–14 s. 6 d.

Ex. 3. What is a Lease of 142 l. 6 s.– the yeare, for 7 yeares, worth in ready money? *Ans.* 739 l.–11 s.–3 d.

1. 7'5361 :: 6'5167

Op. Ex. 1.) 6'5167

53.
452.
0733. (49'1084
37680.
452166.
49'1084

1. 31'0341 :: 65167
6'5167

217.
1862.
03103.
155170.
1862046.
202'2398

Op.

10. 74'6147 :: 52'5
52'5
3730735
1492294 (391'7262.
3730735
391'7262

100. 5.20'637 :: 14 '05.
Op. 3. Ex.) 142'05
26.
1041.
30825. (739'56
52064.
739'56

The use of the fifth table, Prop. 5. to find out the annuity or lease to continue for any years, half-yeares, or quarters, any summe ready money will buy.

Ex. 1. One hath 78 l.–5 s. to bestow of an Annuity, and he would have it paid yearly for 17 years what must that annuity be? *Ans.* 8 l.–11 s. 6 d.

Another hath 428 l. to bestow of a halfe-yearly annuity for 12 yeares and a halfe, what must that annuity be? *Ans.* 27 l.—3 s.—6 d.

Another hath 3489 l. to bestow of a lease for [illegible] yeares, but he would have quarterly rents, what must those rents be? *Ans.* 84 l.– 11 s.–7 d.

Op

Op. 1. Ex. 10. 10.963 :: 78'25.

7.8'25

54.

219.

8770. (8'5784

76741.

85784

100. 6'3442 :: 428. 27'1745

Op. 2. Ex.

4'28

5079.

12698. (27'1745

253968.

27'1745.

1000. 24'242 :: 3484. 84'5802

24'242

648.

13956.

6978

13956

6978 +

CHAP.

Chap. XVI. Of Fractions.

Section 1. *Of finding the greatest common measure of two Numbers.*

1. A Number is said to be prime, which cannot be measured by any number of multitude: such are 2. 3. 5. 7. 11. 13. 17. 19, &c. which no number of multitude (that is) above an unite, can divide evenly, but there will be a Remainer, and therefore are called Prime Numbers.

2. A number is said to be compound, when it may be divided by a number of multitude; as 4. 6. 8. 10. 12, &c. are compound numbers, because they may every of them be divided by a number of multitude, as 8 may be evenly divided by either 2 or 4.

3. Numbers may be said to be prime amongst themselves, when no one number of multitude can measure them both: as 8. and 9. 14. and 15. 21 and 34. &c.

4. Numbers are said to be compound amongst themselves, when one number of multitude will divide or measure them, and the number so measuring them, then is called their common measure; and if it be the greatest that can be, it is called their greatest com

mon meaſure, as 8 and 12 may be meaſured by 2, but their greateſt common meaſure is 4.

5. The greateſt common meaſure of two Numbers is found by a continual Diviſion of the greater by the leſſer, and of the Diviſor by the Remainer; for the firſt Diviſor that divideth the Dividend without a Remayner is the greateſt common meaſure of both the Numbers.

Thus the greateſt common meaſure of 24 *and* 38 *is* 2.

```
24) 38 (1
    24
    ——
  14) 24 (1
      14
      ——
    10) 14 (1
        10
        ——
       4) 10 (2
           8
          ——
        2) 4 (2
           4
           —
           0
```

of 36 and 54 is 18.

```
36) 54 (1
    36
    ——
  18) 36 (2
      36
      ——
```

Thus the greatest common measure of 432 *and* 728 *is* 8.

```
432) 728 (1
     432
     ---
     296) 432 (1
          296
          ---
          136) 269 (2
               272
                    24) 136 (5
                        120
                        ---
                        16) 24 (1
                            16
                            --
                            8) 16 (2
                               16
```

6. And therefore all Fractions whatsoever (if possible) ought to be reduced into their least tearmes before you institute any worke in them as $\frac{6}{54}$ by dividing them by 18 become $\frac{2}{3}$, and so likewise $\frac{432}{738}$ by dividing them by 8, become $\frac{54}{91}$, and have the same value that the first Fraction had for as 432. 728 :: 54 91.

7. Therefore if the tearmes of a Fraction be incommensurable together ; then such a Fraction is in his least tearmes already.

8. If both the Tearmes have outwardly Cyphers,

I meane towards the right hand ſuch a Fraction may be much abridged ſpeedily ; by cutting off a like Number of Cyphers from them both.

Thus $\frac{200}{300}$ *wil be forthwith* $\frac{2}{3}$ *ſo* $\frac{150}{300}$ *wil be* $\frac{15}{30}$ *or* $\frac{1.}{2.}$

9. Even Numbers may be much abridged by halfeing of both as long as they will be halfed ; and ſo may any Number which appeare preſently to the eye, to be equally diviſible or commenſurable by any other Number.

Sect. 1. *Of the Reduction of Fractions.*

1. In Fractions as the Denominator is in the Numerator, ſo one whole is to the parts ſignified by the Fraction ; as for example in the Fraction $\frac{2}{3}$ of a ſhilling, at 3 is to 2, ſo 12d. is to 8d. which is two third parts of a ſhilling.

2. Hence it followeth neceſſarily, that the Tables of the ſame Fraction may be infinite in a number, ſo that they ſtil beare the ſame proportion one to another, as in the example before $\frac{2}{3}$ it may be expreſſed by $\frac{2}{3}$. $\frac{6}{9}$. $\frac{8}{12}$. $\frac{10}{15}$. $\frac{12}{18}$. $\frac{14}{21}$. $\frac{20}{30}$. $\frac{200}{300}$. &c. and ſo infinitely , every one of which Fractions is but ſtil two thirds.

3. Therefore alſo it followeth that if the Numerator be leſſer then the Denominator , that then the parts ſignified are leſſe then one whole, be the tearms

never ſo big, as in $\frac{1379}{5797}$, and this is a Fraction properly ſo called, or a proper Fraction.

4. Therefore alſo if the Numerator be equal to the Denominator, then the Fraction is equal to one whole, as $\frac{2}{2}$. $\frac{3}{3}$. $\frac{20}{20}$, &c. every one of which is equal to one whole, and ought to be expreſſed by 1.

5. Therefore alſo if the Numerator be greater then the Denominator, then the Fraction is greater then the whole, as $\frac{379}{235}$. And theſe two laſt kindes are improperly called Fractions, becauſe they doe include in them one or more Integers.

6. To reduce a whole Number into an improper Fraction of any Denomination given, is done by multiplying the Number by the Denominator, and ſetting the Product over the Denominator in forme of a Fraction; as to reduce 5 whole into halfes, it is thus done $5 \times 2 = 10$ that is $\frac{10}{2}$. ſo 7 into 3 parts, it is $\frac{21}{3}$, and 6 into quarters is $\frac{24}{4}$.

7. Therefore if a mixt Number (*viz.* Integers and a Fraction) be propounded to be reduced into an improper Fraction of the ſame Denomination with the Fraction annexed, the whole number muſt be multiplyed by the Denominator of the Fraction, and unto the Product the Numerator muſt be added, and ſo the ſumme will be the Numerator of the improper Fraction required.

As for Example let $35\frac{3}{5}$ a mixt number, be reduced into an improper Fraction of the ſame Denomina-

tion

tion with $\frac{3}{5}$ it wil be thus $35 \times 5 = 175 + 3 = 178$ and the improper Fraction wil be $\frac{178}{5}$, which is the ſame in value with $75\frac{3}{5}$.

8. Therefore alſo on the contrary, if an improper Fraction propounded be required to be reduced into the Integers or whole Numbers contained in it, the Numerator muſt be divided by the Denominator, and the Quotient wil ſhew the Integers, and the Remaynder (if any be) the odde parts.

Example, let $\frac{10}{2}$ be reduced into Integers, it will be 5 Integers thus $2)\,10\,(5$. ſo $\frac{178}{5}$ is reduced into $35\frac{3}{5}$ thus $5)\,178\,(35\frac{3}{5}$.

9. Two Fractions of divers Denominations are to be reduced to the ſame Denomination their values ſtill remaining thus, *viz.* divide both Denominators by their greateſt common meaſure, by their Quotients multiply the tearmes of both Fractions alternately, and the Products will be two Fractions reduced to the ſame Denomination: Example, let $\frac{5}{12}$ and $\frac{7}{18}$ be Fractions of divers Denominations (*viz.* their Denominators be not the ſame) they will be thus reduced by the greateſt common meaſure of the Denominators 12 and 18, *viz.* 6.

15 14

$$6)\ \frac{5}{12}\quad \frac{7}{18}\quad \frac{15}{36}\quad \frac{14}{36}$$

$$\underbrace{3 \quad 2}_{36}$$

The two Fractions reduced to the ſame Denomination 36. where $\frac{15}{36} = \frac{5}{12}$ and $\frac{14}{36} = \frac{7}{18}$.

21 10

So likewiſe $1)\ \frac{3}{5}\quad \frac{2}{7}\quad \frac{21}{35}\quad \frac{10}{35}$

$$\underbrace{7 \quad 5}_{35}$$

The Fractions reduced to the ſame Denomination 35. where $\frac{21}{35} = \frac{3}{5}$ and $\frac{10}{35} = \frac{2}{7}$.

10. If there be many Fractions of unlike Denominations, whether they be Compound or Prime, they are to be reduced to one Denomination thus, *viz.* Multiply all the Denominators together for the common Denominator, and multiply every Numerator in every Denominator (except his owne) ſo the ſeverall Products ſhall be for the ſeverall new Numerators for every Numerator.

Thus

$$\text{Thus } \frac{1}{2} \text{ and } \frac{2}{3} \text{ and } \frac{3}{4} \text{ and } \frac{4}{5} \text{ reduced thus } \frac{\overset{60.\ 80.\ 90.\ 96.}{\frac{1}{2}\ \ \frac{2}{3}\ \ \frac{3}{4}\ \ \frac{4}{5}}}{120}$$

The common Denominator being 120. for $\frac{60}{120} = \frac{1}{2}$. $\frac{80}{120} = \frac{2}{3}$ $\frac{90}{120} = \frac{3}{4}$ and $\frac{96}{120} = \frac{4}{5}$.

11. Yet if any of the Denominators be compounds together, sometimes the tearmes may be lessened thus, multiply the Denominator with the greater compound, onely rejecting the lesser for the common Denominator, then dividing the common Denominator by every particular Denominator, multiply that Quotient by his owne Nunerator, and place that Product for the new Numerator.

$$\text{Thus, } \frac{\overset{30\ \ 40\ \ 45\ \ 48}{\frac{1}{2}\ \ \frac{2}{3}\ \ \frac{3}{4}\ \ \frac{4}{5}}}{60}$$

Because 2 and 4 in the Denominators are compounded, I reject the lesse 2, and multiply $3 \times 4 \times 5 = 60$, for a new Denominator; Then I divide 60 by $2 = 30$, which I multiply by 1, and set 30 over 1. so dividing 60 by 3, the Quotient is 20, which multiply by $2 = 40$, for the Numerator over 2. &c And now having these Fractions to a like sirname or Denomination with the former, they may be easily applyed to Addition or Substraction, as shall after appeare.

12. If Fractions of Fractions (or particulars) be proposed, they are likewise to be reduced to a new Denominator thus, *viz.* Multiply the Numerators each into other for new Numerators, and likewise Denominators into Denominators, for new Denominators.

$$\text{Thus } \frac{\overline{\tfrac{3}{4} \text{ of } \tfrac{2}{3} \text{ of } \tfrac{6}{7} \text{ of } \tfrac{7}{9}}^{\,252}}{756} \text{ that is } \frac{252}{756}$$

And Fractions of Fractions, when they are reduced to one Denominator, they are applyable to Addition or Substraction with any other Fraction, whether of like Base or unlike, for then they may be reduced according to Rule 9 of this Sect.

11. If you will know what the Fraction of any whole thing signifieth in common parts, you must divide the whole thing in common parts by the Denominator, and the Quotient must be multiplyed by the Numerator: as for Example:

How much is the $\frac{5}{12}$ of a Crowne? Heere the Crowne is in common parts 60 pence; but the Fraction requireth of them $\frac{5}{12}$ that is five twelfth parts: Divide 60 by 12, the Quotient is 5. and 5 multiplyed by the Numerator, 5 is 25, which sheweth that $\frac{5}{12}$ of a Crowne or 60 pence is 25 pence, or two shillings one penny.

Sect. 3.

Sect. 3. *Of Addition and Substraction of Fractions.*

1. First the Numbers proposed to be added, or substracted are either, Fractions with Fractions, Fractions with whole numbers or mixt, or mixt with whole numbers or mixt, and in all these observing the Rules in the last Section, you are to reduce the proposed quantities to one Denomination, and after that according to the signe of Addition + or Substraction — to adde together or substact the Numerators.

Examples of Fractions, with Fractions of the same Denomination.

$+ \frac{1}{5} = \frac{3}{5}$ or $\frac{1}{5}$. $\frac{2}{7} + \frac{4}{7} + \frac{5}{7} = \frac{11}{7}$ or $\frac{1}{7}$. $\frac{54}{60} + \frac{13}{60} = \frac{69}{60}$ or $\frac{41}{60}$

Examples of Fractions with Fractions of unlike Denomination, reduced by Rule 9, of the last sect.

6 + 5   10 12

3) $\frac{2}{3} + \frac{5}{9} = \frac{11}{9}$ or $\frac{1}{9}$.   1) $\frac{2}{3} + \frac{4}{5} = \frac{22}{15}$ or $\frac{2}{15}$.

3   1   5   3

9   15

$$\begin{array}{c} 15 \cdot 14 \\ \text{So } 6)\ \frac{5}{12} + \frac{7}{18} = \frac{29}{36} \text{ or } \frac{1}{36} \\ \underbrace{3 \quad 2}_{36} \end{array}$$

Examples of Fractions with whole Numbers.

$5 + \frac{2}{3}$ that is $4\frac{3}{3} + \frac{2}{3} = 4\frac{5}{3}$ or $4\frac{2}{3}$. where you see I take 1 from the 5 and put in an improper Fraction, that is $\frac{3}{3} = 1$.

$8 + \frac{17}{19}$ that is $7\frac{19}{19} + \frac{17}{19} = 7\frac{36}{19} = 8\frac{17}{19}$ or $7\frac{2}{19}$.

Examp. of mixt Numbers with Fractions.

Of like bases. $2\frac{3}{5} + \frac{2}{5} = 2\frac{5}{5}$ or $2\frac{1}{5}$. but if the latter Fraction be greater then the Fraction of the mixt number you must substract 1 from the Integer, and turne it into an improper Fraction with the first thus

Of like base. $2\frac{3}{5} + \frac{4}{5}$ added is $2\frac{7}{5}$ or $3\frac{2}{5}$ but sub. you must say $1\frac{8}{5} - \frac{4}{5} = 1\frac{4}{5}$.

$$\begin{array}{c} 8 \quad 15 \\ \text{Of unlike Bases } 3\frac{2}{5} + \frac{3}{5} \\ \underbrace{4 \quad 5}_{20} \end{array}$$

Reduced is $3\frac{8}{20} + \frac{15}{20}$ added is $3\frac{23}{20}$ or $4\frac{3}{20}$ but substracted thus $2\frac{28}{20} - \frac{15}{20}$ $2\frac{13}{20}$.

Examp. of whole Numbers and mixt.

$7 \pm 4\frac{2}{3}$ added would be $11\frac{2}{3}$ but substracted thus, $6\frac{3}{3} - 4\frac{2}{3} = 2\frac{1}{3}$.

$3 \pm 2\frac{1}{3}$ added would be $5\frac{1}{3}$ but substracted thus, $2\frac{3}{3} - 2\frac{1}{3} = \frac{2}{3}$.

Examp. of mixt Numbers with mixt.

Of like Bases $5\frac{4}{9} \pm 4\frac{3}{9}$ = by Add. $9\frac{7}{9}$ Sub. $1\frac{1}{9}$.
$7\frac{3}{8} \pm 3\frac{5}{8}$ added is $10\frac{8}{8}$ or 11. but sub.
$6\frac{11}{8} - 3\frac{5}{8} = 3\frac{6}{8}$.

8 ... 9

Of unlike Drs. $4\frac{2}{3} \pm 2\frac{3}{4}$ added is $6\frac{17}{12}$ or $7\frac{5}{12}$ or sub.
$3\frac{20}{12} - 2\frac{9}{12} = 1\frac{11}{12}$.

4 3

12

6 5

Againe $4\frac{3}{5} \pm 3\frac{1}{2}$ added is $7\frac{11}{10}$ or $8\frac{1}{10}$ or sub. $1\frac{1}{10}$.

$\underbrace{2 \quad 5}_{10}$

So to adde. $6\frac{3}{4}$ to $10\frac{1}{2}$ and to $14\frac{5}{6}$ by the 11 Rule of

of the last I reduce the Fractions then it will be $6\frac{18}{24} + 10\frac{12}{24} + 14\frac{20}{24} = 30\frac{50}{24} = 32\frac{2}{24}$.

18. 12. 20.
$\frac{3}{4}$ $\frac{1}{2}$ $\frac{5}{6}$
24

8 ... 9

So to sub. $4\frac{3}{4}$ from $6\frac{2}{3}$ thus $6\frac{2}{3} - 4\frac{3}{4}$ Then $5\frac{20}{12} - 4\frac{9}{12} = 1\frac{11}{12}$.

4 3

12

Sect. 4. *Of Multiplication of Fractions.*

1. In Multiplication and division of Fractions; make mixt numbers improper Fractions, and make whole numbers like Fractions by subscribing an unite.

2. If the Byas tearmes be compounded, bring them to their least tearmes, and the worke will be in the least tearmes, And

Therefore if the Byas tearmes be equall, the other tearmes stand for the Multiplication.

And therefore if the tearmes of any two Fractions be crossewise equall the Product is always an unite.

The Rule for Multiplication of Fractions is, multiply the Numerators together for a new Numerator; and the Denominators for a new Denominator.

Ex

Ex. 1. *How to multiply a mixt number.*

1. *By a mixt number.*
2. *By a whole number.*
3. *By a Fraction.*

(1) $2\frac{3}{4} \times 3\frac{4}{5}$ that is $\frac{11}{4} \times \frac{19}{5} = \frac{209}{20}$.

(2) $2\frac{3}{5} \times 7$. that is $\frac{13}{5} \times \frac{7}{1} = \frac{91}{5}$.

(3) $2\frac{3}{4}\ \frac{2}{5} \times$ that is $\frac{11}{\overset{}{\underset{2}{\cancel{4}}}} \times \frac{\overset{0}{\cancel{2}}}{5} = \frac{11}{10}$

Ex. 2. *How to multiply a whole Number.*

1. *By a mixt.*
2. *By a Fraction.*

(1). $6 \times 3\frac{2}{3}$ that is $\frac{\overset{2}{\cancel{6}}}{1} \times \frac{11}{\underset{0}{\cancel{3}}} = \frac{22}{1} = 22$.

(2) $7 \times \frac{3}{4}$ that is $\frac{7}{1} \times \frac{3}{4} = \frac{21}{4}$.

Ex. 3.

Ex. 3 ***How to multiply a Fraction.***	1.	***By a mixt number.***
	2.	***By a whole number.***
	3.	***By a Fraction.***

(1) $\frac{2}{3} \times 2\frac{4}{5}$ that is $\frac{2}{3} \times \frac{14}{5} = \frac{28}{15}$.

(2) $\frac{3}{4} \times 6$ thus $\frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{1} = \frac{9}{2}$ or $4\frac{1}{2}$.

(3) $\frac{2}{7} \times \frac{3}{5} = \frac{6}{35}$. and $\frac{\overset{1}{\cancel{2}}}{5} \times \frac{3}{\underset{2}{\cancel{4}}} = \frac{3}{10}$.

More Examples.

(1) $\frac{3}{4} \times \frac{6}{9} = \frac{18}{45}$ by the second Rule.

(2) $\frac{9}{12} \times \frac{12}{9} = 1$ by the second Rule.

Secti

Section 5. *Of Division of Fractions.*

1. In Division, if Numerators or Denominators be compounded, reduce them into the least tearmes, and the worke will be in the least tearmes, And Therefore if the Numerators be equal, Cancell them, and set the Denominator of the Divisor over the other in forme of Fraction.

And therefore if the Denominators be equall reject them, and the worke is in the Numerators.

2. The Rule for Division multiply alternly the Numerator of the second with the Denominator of the first for a new Numerator, and the other Numerator and Denominator for a Denominator.

Ex. How to divide a mixt number by { 1 *a mixt number.* 2 *a whole number.* 3 *a Fraction.* }

(1) $3\frac{4}{5})\, 2\frac{3}{4}\, (\frac{55}{76}$ or thus $\frac{19}{5})\, \frac{11}{4}\, (\frac{55}{76}$

(2) $7)\, 6\frac{1}{2}\, (\frac{13}{14}$ or thus $\frac{7}{1})\, \frac{13}{2}\,)\frac{13}{14}$

(3) $\frac{2}{5})\, 2\frac{3}{4}\, (\frac{55}{8}$ or thus $\frac{2}{5})\, \frac{11}{4}\, (\frac{55}{8}$

How to divide a whole number by { 1 *a mixt number.* 2 *a Fraction.* }

(1) $3\frac{2}{3})\, 6\, (\frac{18}{11}$ or thus $\frac{11}{3})\, \frac{6}{1}\, (\frac{18}{11}$

$(2)\ \frac{3}{4})\ 7\ (\frac{28}{3}$ or thus $\frac{3}{4})\ \frac{7}{1}\ (\frac{28}{3}$.

How to divide a Fraction by $\begin{cases} \text{1 \textit{a mixt number.}} \\ \text{2 \textit{a whole number.}} \\ \text{3 \textit{a Fraction.}} \end{cases}$

$(1)\ 2\frac{4}{5})\ \frac{2}{3}\ (\frac{10}{42}$ thus $\frac{14}{5})\ \frac{2}{3}\ (\frac{10}{42}$

$(2)\ 6)\ \frac{3}{4}\ (\frac{3}{24}$ thus $\frac{6}{1})\ \frac{3}{4}\ (\frac{3}{24}$. $(3)\ \frac{3}{4})\ \frac{2}{5}\ (\frac{8}{15}$.

More Examples.

$$\frac{\overset{5}{\cancel{15}})}{\underset{4}{\cancel{16}})}\ \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}}\ (\frac{4}{5}. \qquad \frac{3)}{16)}\ \frac{15}{16}\ (\frac{15}{3}.$$

$$7\frac{2)}{3)}\ \frac{8}{15}\ (\frac{8}{115} \quad \text{thus} \quad \frac{23)}{\underset{1}{\cancel{3}})}\ \frac{8}{\underset{5}{\cancel{15}}}\ (\frac{8}{115}$$

3. How to know the greater Fraction of any two propounded, when a Fraction is divided by another whether it be a true or unproper Fraction, the Quotient doth alwayes expresse the proportion betwixt the Dividend and Divisor.

1. If therefore both the Fractions be equall, both Tearmes of the Quotient will be equal.

2. If the Dividend be the greater Fraction, the Numerator of the quotient must be greater.

1 Thus $\frac{2}{3} = \frac{6}{9}$ Because in Division the tearmes of the Quotient are equall. $\frac{2}{3}) \frac{6}{9} (\frac{18}{18}$.

2. But $\frac{3}{4}$ is greater then $\frac{3}{12}$ Because in Division the Numerator is greater. $\frac{3}{12}) \frac{2}{3} (\frac{36}{12}$.

3. But $\frac{2}{3}$ is lesser then $\frac{3}{4}$ Because in Division the Denominator is greater. $\frac{3}{4}) \frac{2}{3} (\frac{8}{9}$.

Thus you have a plaine way delivered for Fractions; Now come wee to the practice of them in the Golden Rule.

Sect. 5. *The Golden Rule in Fractions.*

1. If any of the tearmes be mixt Numbers, reduce them into improper Fractions.

2. If any of the tearmes be whole Numbers you may make them improper Fractions by subscribing an Unite.

3. If the same figures be in the Dividend, and Divisor, reject or cancell them, and worke Division with the rest.

3. The Rule direct is wrought thus, *viz.* Multiply the Denominator of the first tearme in the Numerators of the second and third tearmes for your

Q Dividend,

Dividend , and multiply the Numerator of the first in the Denominators of the second and third tearmes for the Divisor ; worke out your Division the Quotient gives the answer.

Examp. If $\frac{3}{4}$ yard cost 8s. what shall 2 $\frac{1}{2}$ yards cost? Ans. 1 $\frac{1}{4}$ of a pound.

$$\frac{3}{4} . \frac{2}{5} :: \frac{5}{2} . 1\frac{1}{4}. \qquad \begin{array}{l} 4 \times 2 \times 5 = 40 \;(1\frac{1}{4}. \\ 3 \times 5 \times 2 = 30) \end{array}$$

5. If the second tearme onely be a Fraction, make the first tearme of like Denomination , Reject the Denominators and worke as in whole Numbers.

If 2 yards give $\frac{3}{4}$ l. what shall 7 yards cost ? Answer, 2 $\frac{5}{8}$.

$$\frac{2}{1} . \frac{3}{4} :: \frac{7}{1} . 2\frac{5}{8}. \qquad \text{For } \begin{array}{l} 1 \times 3 \times 7 = 21 \;(2\frac{5}{8}. \\ 2 \times 4 \times 1 = 8 \end{array}$$

Or by the Rule thus $\frac{8}{4} . \frac{3}{4} :: 7 . 2\frac{5}{8}$ By making 2 of the same Denomination.

6. If either the Homogeneall tearmes be of a Fraction, and the other not, reduce it to like denomination , with his like tearme, Cancell the Denominators, and worke as in whole Numbers.

If

If $\frac{3}{4}$ yard. 2s :: (7) or reduced to like Denomination with $\frac{3}{4}$) $\frac{8}{4}$. 18 $\frac{2}{3}$.

If *6* yards or $\frac{24}{4}$. 12 :: $\frac{3}{4}$. 1 $\frac{1}{2}$.

For 24) $\begin{matrix}36\\24\\12\end{matrix}$ (1 $\frac{12}{24}$ = 1 $\frac{1}{2}$.

Sect. 5. *The backward Rule or Rule Inverse in Fractions.*

1. Multiply the Numerators of the first and second tearmes in the Denominator of the third for the Dividend, and the Numerator of the third in the Bases of the first and second tearmes for your Divisor; finish Division heereby the Quotient gives the answer: Examp. If 3 men doe a worke in *6* $\frac{1}{2}$ of houres, in how many houres shall 12 men doe it in? Answer, 1 $\frac{5}{8}$ houres.

$\frac{3}{1}$. (*6* $\frac{1}{2}$) $\frac{13}{2}$ :: $\frac{12}{1}$. 1 $\frac{5}{8}$. 3*13*1 = 39 (1 $\frac{15}{24}$ or 1 $\frac{5}{8}$.
12*1*2 = 24)

If $6\frac{1}{4}$ (i.e.) $\frac{25}{4}$ oz. $6\frac{1}{2}$ (i.e.) $\frac{13}{2}$: : $2\frac{1}{3}$ (i.e.) $\frac{7}{3}$, $17\frac{33}{56}$

$$\begin{array}{lll} 25\times13\times3 = & & 975\ (17\tfrac{33}{56}. \\ 7\times4\times2 = & 56) & \end{array}$$

If a two penny loafe of bread weighed 6l 3 oz. when a bowle of wheate cost 6s. 6d. what is a bowle worth when a two penny loafe of bread weigheth but 2l. 4. oz? Answer $17\frac{33}{56}$ shillings.

$\frac{25}{4}$. $\frac{13}{2}$: : $\frac{7}{3}$. $17\frac{33}{56}$.

$$\begin{array}{lll} 25\times13\times3 = & & 975\ (17\tfrac{33}{56}. \\ 7\times4\times2 = & 56) & \end{array}$$

Sect. 6. *To worke the double Golden Rule in Fractions.*

1. The worke of the Compound Rule direct if any of the tearmes be Fractions, or Fraction like expressed is thus; multiply the 2 first Denominators in the three last Numerators for Dividend, and multiply the 2 first Numerators in the three last Denominators for Divisor: And after that finish your Division.

Examp. If 33l. - 6s. - 8d. in $\frac{3}{4}$ of a yeare bring $2\frac{1}{2}$l. gaine, what shall 400l. stocke bring in $\frac{1}{4}$ of a yeare? Answer, 10l.

33

$$33\tfrac{1}{3} \text{ or } \tfrac{100}{3}\cdot\tfrac{3}{4}\cdot\tfrac{5}{2} \qquad 3\times4\times5\times400\times1 = 24000\,(10$$
$$\tfrac{400}{1}\cdot\tfrac{1}{4}\cdot \qquad 100\times3\times2\times1\times4 = 2400)$$

2. The worke of the Compound Rule converse in Fractions is thus, multiply the Numerators of the first, second, and fifth in the Denominators of the third and fourth tearmes for your Dividend, and multiply the Denominators of the 1, 2, and fifth tearmes in the Numerators of the third and fourth for your Divisor, finish Division, &c.

Examp. If 400l. stocke in $\frac{1}{4}$ of a yeare yeeld 10l. Interest, in what time will 33l. 6s. - 8d. gaine or yeeld 2l. - 10s? Answer, $\frac{3}{4}$.

$$\tfrac{400}{1}\cdot\tfrac{1}{4}\cdot\tfrac{10}{1} \qquad 400\times5\times3 = 6|000 = \tfrac{3}{4}.$$
$$\tfrac{100}{3}\cdot\ \cdot\tfrac{5}{2} \qquad 4\times2\times10\times100 = 8|000$$

3. The worke of the Compound Rule discending when any of the tearmes are Fractions, is thus multiply the Numerators of the 1, 3, and 5 in the bases of the 2 and forth for yonr Dividend, and multiply the bases of the 1, 3, and 5 in the Numerators of the second and fourth tearmes for your Divisor, finish Division, &c.

Evamp. If $\frac{3}{4}$ Duccats countervaile $\frac{2}{3}$ Rose Nobles, and $\frac{4}{5}$ Rose Nobles countervaile 2 Crownes, how many Duccats are equall to 20 Crownes? Answer 9.

Duc. R.N.

RN $\frac{3}{4}$ . . $\frac{2}{3}$ Cr. $\frac{2}{1}$ $3 \times 4 \times 20 \times 3 = 720\ (9.$

$\frac{4}{5}$ . . $4 \times 5 \times 1 \times 2 \times 2 = 80)$

Cr. 20 . .9 Duc.

4. The worke of the Compound Rule aſcending when any of the tearmes are Fractions, is thus, multiply the Numerators of the ſecond. fourth, and fifth, with the Denominators of the firſt and third for the Dividend, and multiply the Denominators of the ſecond, fourth and fifth with the Numerators of the firſt and third for the Diviſor, finiſh Diviſion, &c.

Examp. If $\frac{3}{4}$ Duccats countervaile $\frac{2}{3}$ Roſe Nobles, and $\frac{4}{3}$ Roſe Nobles countervaile 2 Crownes, how many Crownes will countervaile 9 Duccats? Anſw. 20.

$\frac{3}{4}$ Duc. . $\frac{2}{3}$ R:N: $2 \times 2 \times 9 \times 4 \times 5 = 720\ (20$

$\frac{4}{3}$ RN. . $\frac{2}{1}$ Cr. $3 \times 3 \times 4 = 35)\ ^{72}.$

$\frac{9}{1}$ Duc. . 20 Cr.

Sect. 7. *The Rule of Fellowſhip in Fractions.*

Examp. Foure men, *viz.* *A, B, C, D,* take a Prize worth 8190l. whereof *A* ſhould have $\frac{1}{3} + \frac{1}{10}$ of the value, *B* $\frac{1}{4} + \frac{1}{10}$. *C* $\frac{1}{6}$ and *D* $\frac{1}{10}$, what muſt each have for

for his share? First *A* and *B* shares severally, *A* share will be $\frac{13}{30}$, *B* share $\frac{7}{20}$. Then reduce $\frac{13}{30}+\frac{7}{20}+\frac{1}{6}+\frac{1}{20}$ to like Base, so they will be $\frac{26}{60}+\frac{21}{60}+\frac{10}{60}+\frac{3}{60}=\frac{60}{60}$. Then worke for *A*. 60. 8190 :: 26. 3549. *B*. 60. 8190 :: 21. 2866 $\frac{1}{2}$. *C*. 60. 8190 :: 10. 1365. *D*. 60. 8190 :: 3. 499 $\frac{1}{2}$. Therefore *A* must have 3549l. *B* 2866l. 10s. *C* 1365l. *D* 499l. 10s. = 8190.

Sometimes in this Rule as was before noted, divers questions impossible to be answered may be proposed, as in this Example.

Foure men, *viz.* *A*. *B*. *C*. *D*. agree to divide 600 Crownes amongst them, so that *A* shall have $\frac{2}{3}$ thereof more by 9 Crownes, *B* shall have $\frac{3}{5}$ more by 8 Crownes, C shall have $\frac{5}{6}$ + 7 Crownes, and *D* shall have $\frac{7}{8}$ more by 6 Crownes, what shall each have?

For Answer because $\frac{2}{3}+\frac{3}{5}+\frac{5}{6}+\frac{7}{8}$ makes by Reduction $\frac{357}{120}$, and $9+8+7+6=30$ maketh $\frac{6}{120}$, for 600 represented by 120, being divided by 30, quoteth 20, and 120 divided by 20, quoteth 6, that is $\frac{6}{120}$.

Now I say because $\frac{357}{120}+\frac{6}{120}$ maketh $\frac{363}{120}$, which amounteth to 3 Integers and $\frac{1}{40}$ more, all which ought only to be but $\frac{120}{120}$, or one value, therefore there cannot be a Division made according to the Position of the question: But when the *Overplus* that is 30 Crownes are deducted out of the totall 600 Crownes, the Division must be made according to the Proportion, which the Numerators of the Fractions reduced beareth each to other.

$$\frac{80+32+100+105}{\frac{2}{3}+\frac{3}{5}+\frac{5}{6}+\frac{7}{8}} = 357.$$
$$120$$

Thus as oft as *A* shall have 16 Crownes. *B* 135. and *C* 200, and *D* 210 by making the totall of their shares 705 the generall Antecedent, 570 that is 600 -30 the generall consequent, and every Partners part the particular Anteceedent in the Golden Rule thus.

A. 357. 570 :: 80. $127\frac{261}{357}$. *B*. 357. 570 :: 72. $114\frac{342}{357}$. *C*. 357. 570 :: 100. $159\frac{237}{357}$. *D*. 357 570 :: 105. $167\frac{231}{357}$.

And now
$$A\ 127\tfrac{261}{357}+9=136\tfrac{261}{357}.$$
$$B\ 114\tfrac{342}{357}+8=122\tfrac{342}{357}.$$
$$C\ 159\tfrac{237}{357}+7=166\tfrac{237}{357}.$$
$$D\ 167\tfrac{231}{357}+6=\ 67\tfrac{231}{357}.$$

All which summes are equall to 600 which is the proofe of the question.

Sect. 8. *The consideration of the* Ratio *of two numbers in quantity, and of the Addition, and Substraction thereof.*

1. The simple reason betwixt two Numbers is found by dividing the Antecedent by the Consequent, and therefore they are expressed in forme of a Fraction,

raction, as 2 to 2 set thus $\frac{2}{2}$, is the reason of equalli-
y, because the first Antecedent, or Numerator, be-
ng divided by the latter Consequent, or Denomina-
or, is contained equally once in the same; so 2 and
or $\frac{2}{1}$ is of inequallity, and double because 2 divi-
ed by 1 is contained twice therein.

2. The Addition of these Ratios is as though they
vere Fractions and to be multiplyed, as if you were
adde $\frac{1}{2}$ to $\frac{3}{4}$, that is, if the proportion of 1 to 2 be
be added to the proportion of 3 to 4, multiply
em as though they were Fractions, and the Pro-
ucts shewes the proportion augmented to be $\frac{3}{6}$, or as
is to 6. $\frac{1}{2}$ $\frac{3}{6}$ = $\frac{3}{6}$, so $\frac{3}{4}$ to $\frac{8}{5}$ added is $\frac{24}{20}$, so $\frac{4}{3}$ $\frac{9}{8}$ = $\frac{36}{24}$. so
added to $\frac{3}{2}$ makes $\frac{24}{6}$ = $\frac{4}{1}$. ***Ex. Euclid. lib.*** 8. Pro. 5.

3. The substraction of one ***Ratio*** from another,
done by dividing the ***Ratios*** as though they were
ractions; For if you divide the ***Ratio*** from which
e substraction is to be made by the ***Ratio*** to be sub-
racted, you have done that which is desired: As if
e ***Ratio sesquialtera*** $\frac{3}{2}$ be to be substracted from the
ouble ***Ratio*** $\frac{2}{1}$, the Remaynder will be $\frac{4}{3}$ ***sequitertia.***
) $\frac{2}{1}$ ($\frac{4}{3}$. So $\frac{19}{7}$ out of $\frac{11}{2}$ resteth $\frac{209}{14}$.

The proofe of Substraction in this kinde is thus,
the ***Ratio*** found be added to the ***Ratio*** subducted,
e make up the ***Ratio*** from which the subduction
as made, as in the example where $\frac{3}{2}$ was substract-
d from $\frac{2}{1}$, the Remaynder was $\frac{4}{3}$, which $\frac{4}{3}$ added to $\frac{3}{2}$
akes $\frac{2}{1}$. and so likewise may you examine Addition
y Substraction.

1. Examples

1. *Examples of Addition of* Ratios.

The Proportion of a penny to a farthing is as 4 to 1, the proportion of a shilling to be a penny is as 12 to 1, what are these proportions added? Answer, $\frac{4}{1}\,\frac{12}{1} = \frac{48}{1}$, which is the proportion of 1 shilling to one farthing.

2. Examp. A horse beares a weight, but he may easily beare three times as much, and another horse beares a weight *t*, but he may well beare three quarters as much as he doth; the question how much these two might draw or beare betwixt them? Add the *Ratios* $\frac{3}{1}$ and $\frac{3}{4}$ makes $\frac{6}{4}$ or $1\frac{1}{2}$, and therefore they might well draw betwixt them as much, and half as much as they did.

3. Examp. A Fountaine runs at three pipes, if the first pipe run it would fill the Conduct in 2 houres, if the second run it would doe the same in 5 houres, if the third in 12 houres: It is desired to know that if the same run all at once, in how many houres will it fill the Conduct? For answer the proport. of the first is $\frac{1}{2}$, the second $\frac{1}{5}$, the third in $\frac{1}{12}$. the Fractions $\frac{1}{2} + \frac{1}{5} + \frac{1}{12}$ reduced make $\frac{30+12+5}{60}$.

Therefore I conclude that 47 Conducts will be filled in 60 houres, and therefore one Conduct would be filled if they all run, in one houre and 13 minutes

Examp

Examples of Substraction of Ratios.

Examp. 1. There are two workemen, one doth he worke in 30 dayes, but being joyned in worke ith the other he performes the worke in 12 dayes, is desired to know the ability of the other workean, and in how many dayes he could performe the me worke? For answer I substract the *Ratio* of $\frac{1}{30}$ t of $\frac{1}{12}$ reduced is $\frac{4-10}{120} = \frac{6}{120}$ or $\frac{1}{20}$. Therefore I conude the latter man could have done it in 20 dayes himselfe.

Examp. 2. A Fountaine hath two pipes, that hich fils the Conduit is the greater, that which npties is lesse; the greater will fill the Conduit in houres, the lesse will empty it in 22 houres: It is sired that seeing the proportion of the filling the onduit is greater then the evacuation, both the pes running, in what time the Conduit will be fil-d? For answer I substract the proportion $\frac{1}{8}$ out of resteth $\frac{7}{88}$, and therefore the Conduit will be filled 12 $\frac{4}{7}$ houres.

$$\left(2 \quad \frac{\frac{1}{8} \quad \frac{1}{22}}{11 \quad 4}\right| \quad \frac{4}{88} - \frac{11}{88} = \frac{7}{88}$$

These precepts doe extraordinarily conduce both the mixture of Medicines before handled, and to e speculative part of Musick; for the *Diapente* being

being *sesquialter*, Diatesseron *sesquitertia*, Diapaso[n] *dupla*, Diapason with the Diapente triple, and the Tone *sesquioctava*: From hence the Musitians have found out that the Diapason is made of the Diatesseron added to the Diapente, and that the Diapente is made of the Diatesseron, added to the Tone, &c. as also to the powers of all sort of Machinations or forcible Instruments.

Sect. 9. *Of the Multiplication, and* Division *of* Ratios.

1. Place the Ratio proposed to be multiplyed so many times over, as there are Unites in the multiplyer, and multiply the Denominators for a new Denominator, & the Numerators for a new Numerator, if you were to multiply the Ratio of $\frac{2}{3}$ by 3. it is $\frac{8}{27}$

Thus $\frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{8}{27}$. so $\frac{4}{2}$ by 2 is $\frac{16}{4} = 4$.

Thus $\frac{4 \times 4}{2 \times 2} = \frac{16}{4}$.

So if you would double $2\frac{1}{4}$, it is $\frac{81}{16}$. $\frac{9}{4} \times \frac{9}{4} = \frac{81}{16}$

2. Hence it appeareth that to double any *Ratio* is but to square it, and to triple it to Cube it &c.

3. And on the contrary to divide any Ratio by 2 is but to take the square roote thereof, and to divide

by 3 is to take the Cube roote, &c. As for examle to take the halfe of 2 $\frac{1}{4}$ or the Ratio $\frac{9}{4}$ or divide it y 2 is to take the ſquare roote thereof, *viz.* $\frac{3}{2}$ that is $\frac{1}{2}$.

These have great uſe in many hard examples, for heſe Proportions doe not onely finde out the Deonſtrations of the Rules of Proportion, but doe exlaine many peculiar and hard Problmes.

As in this out of Ptolomy Almag.

The Diameter of the Sunne to the Diameter of e earth hath the Proportion as 11 to 2, or $\frac{11}{2}$.

The Diameter of the Sun to the Diameter of the oone hath the Proportion as 94 to 5, or $\frac{94}{5}$.

The Diameter of the Earth to the Diameter of the oone hath the Proportion as 17 to 5, or $\frac{17}{5}$. from eſe Proportions, the greateſt and remote bodyes ll under meaſure.

For becauſe Spheres or Globes have the triple proortion to their Diameters, *per vlt. pro. Eu.* 12. herefore triple $\frac{11}{2}$ the proportion which the Sunne th to the Earth, and you have the proportion that e body of the Earth beareth to the body of the nne, the which by the worke will be $\frac{1331}{8}$, or 166 $\frac{3}{8}$, at is the Sunnes body is bigger then the body of the arth 166 times and $\frac{3}{8}$.

$$\tfrac{11}{2} \times \tfrac{11}{2} \times \tfrac{11}{2} = \tfrac{1331}{8}. \text{ and } 8)\,\underset{53}{\overset{1}{1}}\!\!33\,1\,(166\tfrac{3}{8}.$$
$$\underline{51}$$

After the same manner you will finde the Sun [illegible] be greater then the Moone *6644* $\frac{84}{17}$, And lastly th[illegible] Earth to be bigger then the Moone *39* times and [illegible] most $\frac{1}{3}$.

Sect. 10. *Of the severall* Species *of* Ratios.

1. The proportion of two numbers consists eith[illegible] in equallity or unequallity , equallity when bo[illegible] numbers are equall as $\frac{5}{5}$.

2. Inequallity is either greater then equallity as [illegible] or lesse $\frac{3}{5}$.

3. The proportion of inequallity is either prim[illegible] or conjunct.

4. Prime when the Quotient is alwayes unity [illegible] a Fraction, and this is either manifold, superparticu[illegible] lar, or superpertient.

5. Proportion conjunct is when the Quotient ex[illegible] ceedeth an vnite as $\frac{5}{2}$, and this is either manifold super[illegible] particuler, or manifold superpertient ; but these a[illegible] sufficiently explained in most Arithmetick book[illegible] and being of no speciall use , I therefore leave them [illegible] and passe now to the second part of Arithmetick i[illegible] Species.

A Tabl of Compound Interest at 8 per Centum, Per Annum, *from one yeere to* 100; *for yeares, six monthes, or three monthes.*

	1019426		1617701		2567089
	1039230		1649128		2616959
	1059419		1681165		2667797
1	1080000	7	1713824	13	2719623
	1100980		1747117		2772456
	1122368		1781058		2826315
	1144172		1815658		2881221
2	1166400	8	1850930	14	2937193
	1189059		1886887		2994253
	1212158		1923543		3052421
	1235706		1960910		3111719
3	1259712	9	1999004	15	3172169
	1284183		2037838		3233793
	1309131		2077426		3296614
	1334563		2117783		3360656
4	1360488	10	2158925	16	3425942
	1386918		2200865		3492496
	1413861		2243620		3560344
	1441328		2287206		3629509
5	1469328	11	2331639	17	3700018
	1497872		2376954		3771896
	1526970		2423110		3845171
	1556634		2470182		3919869
6	1586874	12	2518170	18	3996019

	4073648		6981517		11955093
	4152785		7117144		12197534
	4233459		7255405		12434430
19	4315701	26	7396353	33	12676049
	4399540		7540038	34	13090133
	4485008		7686515	35	4785344
	4572136		7835838	36	15968171
20	4660957	27	7988061	37	17245635
	4751503		8143242	38	1862[illegible]275
	4843808		8301436	39	20115297
	4937907		8463705	40	21724521
21	5033833	28	8627106	41	[illegible]3462483
	5131623		8794701	42	[illegible]339481
	5231323		8965551	43	27366640
	5332939		9139721	44	29555971
22	5436540	29	9317274	45	319[illegible]0449
	5542153		9498277	46	34474085
	5649818		9682796	47	37232012
	5759577		9870899	48	40210573
23	5871466	30	10062656	49	43427[illegible]18
	5985522		10258139	50	46901012
	6101804		10457419	51	50753741
	6220340		10660571	52	54706044
24	6341180	31	10867669	53	59082523
	6464367		11078790	54	63809123
	6589948		11294013	55	68913851
	6716968		11513417	56	74442690
25	6848475	32	11737082	67	80381124

58	868116[illegible]6	74	297411602	90	1018915089
59	93756546	75	321604530	91	1100428290
60	101257063	76	346900892	92	1188462560
61	109357628	77	374652964	93	1[illegible]83539564
62	118106[illegible]39	78	404625201	94	1386222730
63	127554738	79	436995217	95	1497120548
64	137759[illegible]17	80	471954834	96	161689092
65	148779846	81	509711221	97	174624408
66	160682234	82	55048[illegible]119	98	1885940720
67	173536813	83	594527168	99	20368[illegible]5978
68	187419758	84	642089342	100	2199761256
69	202413338	85	6934[illegible]6489		
70	218606405	86	748933008		
71	236094918	87	808347649		
72	25498[illegible]512	88	871555461		
73	275381112	89	943439897		

2. *A* Table of *Compound Interest discontinued at* 8 per Cent. *from one yeare to* 100, *by yeares, six monthes, and* 3 *monthes.*

	980944		841001		721023
	962250		824974		707283
	943913		809243		693804
1	9259[illegible]6	3	7938[illegible]	5	680583
	908181		778705		667614
	890973		763865		654891
	873994		749309		642312
2	857339	4	735029	6	630169

	618161		360691		210459		122801
	606381		353817		206449		120461
	594823		347075		202515		118165
7	583490	14	340461	21	198656	28	115913
	572371		333973		194870		113705
	561464		327609		191156		111538
	550764		321365		187513		109412
8	540268	15	315241	22	183940	9	107327
	529973		306234		180435		105282
	519874		303351		176997		103276
	509967		297561		173624		101308
9	500249	16	291890	23	170315	30	99377
	490716		286328		167069		97483
	481364		280872		163886		95626
	472199		275519		160763		93804
10	463192	17	270269	24	157699	31	92016
	454396		265118		154694		90263
	445708		260066		151746		88542
	437214		255110		148854		86855
11	428883	18	250249	25	146017	32	85200
	420709		245480		143235		83576
	4[illegible]2692		240802		140506		81984
	484828		236213		137828		80421
12	397114	19	231712	26	135302	33	78888
	389546		227296		132625	34	73045
	382123		222965		130098	35	67634
	374841		218716		127618	36	62624
13	367698	20	214548	27	125187	37	57985

38	53690	54	15672	70	4574	86	1335
39	49713	55	14510	71	4235	87	1236
40	46030	56	13436	72	39[illegible]2	88	1145
41	4162	57	12440	73	3631	89	1060
42	39464	58	11519	74	3362	90	981
43	36541	59	10666	75	3113	91	909
44	33834	60	9875	76	2883	92	841
45	31328	61	9144	77	2669	93	779
46	29007	62	8467	78	2471	94	721
47	26858	63	7840	79	2288	95	668
48	24869	64	7259	80	2119	96	618
49	23026	65	6721	81	1962	97	572
50	21321	66	6223	82	1816	98	538
51	19742	67	5762	83	1682	99	491
52	18279	68	5336	84	1557	100	454
53	16925	69	4940	85	1442		

3. *A Table of Compound Intereſt for Penſions or Annuities, for yeares, and ½ yeares.*

1	1000000	9	12487557	17	33750226
2	2080000	10	14486562	18	37450244
3	3246400	11	16645487	19	31446263
4	4506112	12	18977126	20	45761964
5	5866600	13	21495296	21	50422921
6	7335929	14	24214020	22	55456755
7	8922863	15	27152114	23	60893295
8	10636627	16	30324283	24	66764759

25	731059400	53	726031551	81	635889026
26	99954415	54	785114075	82	686860145
27	87350768	55	848923201	83	741908960
28	95338829	56	9 7827057	84	801361677
9	103965436	57	992264022	85	86557061
30	113283211	58	1072645143	86	9349162601
31	123345868	59	1159456755	87	10098095609
32	134213537	60	1253213796	88	10906943257
33	145950620	61	1354470359	89	1178049871
34	158626670	62	1463827988	90	127239386
35	172316803	63	1581934227	91	137428537
36	187102147	64	1709488965	92	148432820
37	203070319	65	1847248083	93	116031745
38	220315945	66	1996027929	94	11731528
39	238941221	67	2156710163	95	127239388
40	259056518	68	2330246577	96	201986274
41	280781040	69	2517667735	97	21815517 59
42	304243523	70	2720080073	98	23561759 00
43	329583005	71	2938686479	99	25447699 7
44	356949645	72	3174781398	100	274845157
45	386505617	73	3429763910		100000
46	418426067	74	3705145023	1	203923
47	451900152	75	4002556624		311923
48	490132164	76	4323761154	2	424159
49	530342737	77	4670662047		540799
50	573770156	78	5045315010	3	662015
51	620671769	79	5449940711		787986
52	671325510	80	5886935428	4	918900

	10549489		80365698		285429419
5	11963351	19	84514183	33	297626954
	13432679		88834184		310303004
6	14959650	20	93319192	34	323476341
	16546524		97980149		337166475
7	18195653	21	102823958	35	351393679
	19909477		107857792		366179023
8	21690535	22	113089105	36	381544404
	23541465		118525656		397512576
9	25465009	23	124175464	37	414107186
	27464013		130046928		431352812
10	29541439	24	136146732	38	449274992
	31700365		142489913		467930268
11	33943985	25	149079861	39	487256222
	36275614		155928336		507371520
12	38698735	26	163045481	40	528275950
	41214905		170441834		550000472
13	43833865	27	178128349	41	572577257
	46553488		186116411		596039759
14	49379803	28	194417848	42	620432668
	52316997		203044954		645762149
15	55369418	29	212010506	43	672095711
	58451587		221327781		699462352
16	61838203	30	231000577	44	727902599
	65264145		241073234		757458570
17	68824489	31	251530654	45	788174037
	72524507		262398324		820094487
18	76369679	32	273692337	46	853767191

	887741276		1039703024
47	923567997	49	1081491077
	960799809		1124918496
48	999492451	50	1170049594

4. *A Table for Pensions or Annuities discontinued for yeares, six monthes, or three months.*

	1000000		25628438		61815692
	2019426		27126309		64016558
	3058657		28653280		66260179
1	4118076	6	30209915	11	68547385
	5198076		31796789		70879024
	6299057		33414490		73255959
	7421425		35063619		75679069
2	8565598	7	36744783	12	78149252
	9731998		38458608		80667422
	10921057		40205726		83234511
	12133216		41986784		85851471
3	13368922	8	43802442	13	88519268
	14628634		45653373		91238892
	15912818		47540[illegible]60		94011348
	17221949		49463603		96837664
4	18556512	9	51424714	14	99718886
	19917001		53423719		102656080
	21303920		55461557		105650333
	22717782		57538983		108702754
5	24159109	10	59656767		111814473

	114986642		233811682		437456919
	118220435		239353826		446955197
	121517052		245003654		456627993
16	124877707	23	250765229	30	466508892
	128303650		256634693		476571549
	131796146		262620219		486829689
	135356490		268722023		497287108
17	138986000	24	274942364	31	507947680
	142686018		281283545		518815349
	146457914		287747912		529894140
	150303086		294337861		541188153
18	154222956	25	301055829	32	552701570
	158218975		307904304		564438653
	162292624		314885822		576403747
	166445409		322002966		588601282
19	170678869	26	329158572	33	601035772
	174994570		336654725		613711822
	179394110		344194764		626634123
	183879118		351881279		639807461
20	188451254	27	359717118	34	653236710
	193113211		367705179		666926844
	197863715		375848421		680882930
	202707524		384149858		695110134
21	207645431	28	392612563	35	709613723
	212679265		401239670		724399068
	217810888		410034371		739471640
	223042202		418999923		754837021
22	228375142	29	428139644	36	770500897

	786469069		1179740195		1757584502
	802747448		1203658473		1792728300
	819342098		1228041401		1828554840
37	836259046	42	1252898005	47	1865077326
	853504671		1278237487		1902309338
	871085370		1304069227		1940264640
	889007499		1330402789		1978957282
38	907277845	43	1357247921	48	2018421588
	925903121		1384614562		2058612167
	944890[illegible]21		1412512842		2099603887
	964245176		1440953089		2141391940
39	983978419	44	1469945832	49	2183991791
	1004093447		1499501803		2227419210
	1024599515		1529631945		2271690274
	1045503946		1560347412		2316821272
40	1066814477	45	1591659574	50	2362839211
	1088538999		1623580024		
	1110685553		1656120577		
	1133262333		1689293281		
41	1156277712	46	1723110410		

5. ***A Table for Pensions or Annuities discontinued for yeares, and six monthes.***

1	925925	5	3992710	9	6246888
2	1783264	6	4622879	10	6710081
3	2577096	7	5206370	11	7138964
4	3312126	8	5746629	12	7536078

13	7903776	41	11967234	69	12438245
14	8244236	42	12006698	70	12442819
15	8559478	43	12043239	71	12447055
16	8851369	44	12077073	72	12450977
17	9121638	45	12101401	73	12454608
18	9371887	46	12137408	74	12457970
19	9603599	47	12164267	75	12461084
20	9818147	48	12189136	76	12463966
21	10016803	49	12212163	77	12466635
22	10200743	50	12233484	78	12469107
23	10371059	51	12253226	79	12471395
24	10528758	52	12271506	80	12473514
25	10674776	53	12288431	81	12475476
26	10809977	54	12304103	82	12477292
27	10935164	55	12318614	83	12478974
28	11051078	56	12332050	84	12480532
29	11158406	57	12344490	85	12481944
30	11257783	58	12356010	86	12483309
31	11349799	59	12366675	87	12484545
32	11434999	60	12376551	88	12485690
33	11513888	61	12385696	89	12486750
34	11586933	62	12394163	90	12487732
35	11654568	63	12402002	91	12488640
36	1171719	64	12409261	92	12489482
37	11775178	65	12415983	93	12490261
38	11828868	66	12422206	94	12490982
39	11878582	67	12427969	95	12491650
40	11924613	68	12433304	96	12492269

97	12492841		15749922		22174134
98	12493372	13	16117620	27	22299321
99	12493862		16471438		22419782
100	12494317	14	16811899	28	22535696
	962250		17139508		22647234
1	1888176	15	17454749	29	22754561
	2779149		1758091		22847837
2	3636488	16	18049981	30	22957215
	4461462		18330853		23052840
3	5255394	17	18601122	31	23144857
	6019160		18861188		23233399
4	6754190	18	19111437	32	23318599
	7461472		19352240		23400583
5	8142056	19	19583952	33	23479472
	8796947		19806917		23555383
6	9427117	20	20021465	34	23628428
	10033498		20227914		23698716
7	10616988	21	20426570	35	23766350
	11176452		20617726		23831432
8	11718721	22	20801667	36	23894056
	12238595		20978664		24954317
9	12738844	23	21148979	37	24012302
	13220209		21312865		24068099
10	13683402	24	21470564	38	24121790
	14129110		21622311		24173453
11	14557993	25	21768329	39	24223167
	14900686		21908834		24271004
12	15367800	26	22044036	40	24317035

	24361328		24660493		24880389
41	24403949	45	24691821	49	24903415
	24444961		24721967		24925573
42	24484425	46	24750974	45	24946894
	24522400		24778886		
43	24558941	47	24805745		
	24594102		24831589		
44	24627936	48	24856459		

6. *A Table for Pensions discontinued for three monthes.*

	980943		14360613		24195070
	1943194		15067896		24714944
	2887107		15761700		25224911
1	3813033	5	16442284	9	25725160
	4721314		17109898		26215876
	5612287		17764789		26697241
	6486281		18407201		27169433
2	7343620	6	19037370	10	27632628
	8184621		19655531		28086693
	9009595		20261912		28532701
	9818849		20856738		28969915
3	10612681	7	21440228	11	29396798
	11391386		22012599		29819508
	12155251		22574063		30232201
	12904560		23124828		30637029
4	13639590	8	23665096	12	31034143

	31433689		39775649		44648937
	31805812		39998614		44779035
	32180653		40 17330		44906654
13	32548351	20	40431879	27	4503 841
	32909042		40642338		45154642
	33262859		40848787		45275103
	33609934		41051302		45393269
14	33950396	21	41249958	28	45509183
	34284369		41444828		45622887
	34611977		41635985		45734425
	34933343		418 3498		45843838
15	35248585	22	42007439	29	45951166
	35557819		42187874		46056445
	35861161		42364871		46159724
	36158721		42538495		46261032
16	36450512	23	42708811	30	46360409
	36736940		42875880		46457893
	37017812		43039776		46553518
	37393731		43200529		466473 2
17	37563600	24	43358228	31	46739338
	37828719		43511922		46829601
	38088785		43664669		46918143
	38343896		43813523		47004998
18	38594145	25	43959411	32	47090198
	38839625		44102776		47173775
	39030427		44243283		47255759
	39316641		44381110		47336180
19	39548353	26	44516312	33	47415069

	47492454		48965673		49894050
	47568365		49013509		49924195
	47642830		49060435		49953767
34	47715875	40	49106466	46	49982774
	47785728		49151619 8		50011228
	47857816		49195913		50039141
	47926765		49239362		50066521
35	47994399	41	49281983	47	50093377
	48060745		49323792		50119716
	48125826		49364804		50145571
	48189668		49405035		50170923
36	48252292	42	49444499	48	50195792
	48313723		49483211		50220188
	48373984		49521186		50244118
	48433096		49558437		50267592
37	4849108[illegible]	43	49594977	49	50290619
	48547962		49630822		50313207
	48603759		40665983		50335364
	48658493		49700475		50357100
38	48712 83	44	49734309	50	503784[illegible]1
	48764851		49767498		
	48816514		49800055		
	48867193		49831991		
39	48916907	45	49863319		

7. A Table

7. *A Table to find out the value of a Pension or Annuity for yeares and six monthes.*

1	1080000	25	93678	49	81885	73	80291
2	560769	26	92507	50	81742	74	80269
3	388033	27	91448	51	81611	75	80249
4	301921	28	90489	52	81489	76	80231
5	250456	29	89618	53	81376	77	80214
6	216315	30	88827	54	81273	78	80198
7	192072	31	88107	55	81177	79	80183
8	174014	32	87451	56	81089	80	80169
9	160070	33	86851	57	81007	81	80157
10	149029	34	86304	58	80932	82	80145
11	140076	35	85803	59	80862	83	80134
12	132695	36	85344	60	80798	84	80124
13	126521	37	84924	61	80738	85	80115
14	121297	38	84538	62	80683	86	80107
15	116829	39	84185	63	80632	87	80099
16	112988	40	83860	64	80584	88	80091
17	109629	41	83561	65	80541	89	80084
18	106702	42	83287	66	80501	90	80078
19	104127	43	83034	67	80463	91	80072
20	101842	44	82801	68	80429	92	80067
21	99832	45	82587	69	80397	93	80062
22	98032	46	82389	70	80367	94	80057
23	96422	47	82208	71	80340	95	80053
24	94977	48	82040	72	80314	96	80049

97	80045		63492		45097		41048
98	80042	13	62043	27	44844	41	40977
99	80039		60711		44603		40908
100	80036	14	59481	28	44374	42	40842
	1039130		58344		44155		40779
1	529664	15	57291	29	43947	43	40718
	359822		56312		43748		40660
2	274990	16	55401	30	43559	44	40604
	224141		54553		43378		40550
3	190284	17	53760	31	43206	45	40499
	166136		53019		43041		40450
4	148056	18	52324	32	24884	46	40402
	134021		51673		42734		40356
5	122819	19	50487	33	42590	47	40313
	113675	20	49946		42453		40271
6	106076		49436	34	42322	48	40230
	99666	21	48956		42196		40192
7	94188		48502	35	42076	49	40155
	89458				41961		40119
8	85333	22	48073	36	48151	50	40085
	81709		47667		41746		
9	78500	23	47283	37	41645		
	7564		46920		41548		
10	73081	24	46575	38	41456		
	70775		46248		41367		
11	63691	25	45948	39	41282		
	66797		45643		41201		
12	65071	26	45363	40	41123		

8. *A Table to find out the value of a Pension for three monthes.*

	1009427		50876		31823		25747
	514617		49353		31440		25588
	346367		47945		31074		25434
1	262158	7	46641	13	30723	19	25285
	211805		45420		30386		25141
	178181		44298		30063		25001
	154170		43243		29753		24865
2	136172	8	42256	14	29454	20	24733
	122180		41330		29167		24605
	110992		40461		28892		24480
	101844		39643		28625		24359
3	94227	9	38872	15	28370	21	24242
	87785		38145		28123		24128
	82269		37457		27885		24017
	77491		36806		27656		23910
4	73316	10	36189	16	27454	22	23805
	69635		35603		27230		23703
	66366		35047		27014		23604
	63445		34518		26814		23508
5	60819	11	34015	17	26621	23	23414
	58445		33535		26434		23323
	56293		33077		26254		23234
	54326		32640		26079		23147
6	52528	12	32222	18	25910	24	23064

Chap. XVII.

The use of the Logarithmes *in a far more easier manner then formerly, as also generall Rules by them for Compound* Interest, *Annuities, &c. at any rate desired.*

The Canon of Logarithmes *is also in every mans hand; but their perfect use in decimall Fractions knowne to a few; I have therefore amplified with Examples what Mr.* Oughtred *hath briefely delivered in his* Clavis *concerning them and Compound* Interest, *with Annuities, being of singular use, and speedy performance.*

Sect. 1. *Of the* Logarithmes.

1. THe *Logarithme* of any Number is found out of the Canon thus; under the Title N. seeke out the Number proposed, and the number of figures answering under the title of *Log.* is the Logarithme desired.

Examp. The Logarithme of 34 is 1.53148. of 3454 is 3.53832.

2. The first figure of every *Logarithme*, may fitly be tearmed the *Index*, shewing the number and nature of the figures adjoyning; *viz.* how far distant from

unity, as hath been perfectly taught before: Therefore, whether the number proposed be whole, mixt, or Decimals, finde out the *Logarithme* to it, as if it were a whole Number; and then according to the distance from unity of the first figure perfix an *Index* to the Logarithme found, which alwayes differ, as the nature of your number doth, though the Logarithmes may be the same, as in the Examples following.

Numb.	*Logarithmes.*
3571.	3.55279.
35'71	1.55279
3'571	0.55279
'003571.	$\bar{3}$.55279.

Note that the same *Logarithmes* serve, but the *Index* alters according to the nature of the first Figure of the Number; for in the first the Figure 3 is the third from unity; therefore the *Index* of the *Log.* is 3. but in the last it is $\bar{3}$ negative, because Decimals.

3. If the *Logarithme* be given, and the Number answering it be desired; seeke among the Logarithmes where the Index is greatest (neglecting for the most part the Index of the *Log.* proposed) for it, and see downe the Number answering, which must be ordered according to its owne Index, what *Integers*, *Decimals*, or Mixt must be kept.

As for Example, suppose the *Log.* 2.57825 were proposed, to finde its corresponding *Number*, I looke for

or this *Log.* under the Index 4, it my *Canon* give
eave and finde the Number 37866, which according
o the Index 2 is 378'66.

Likewise the Number answering 1.64345 is 44000
hat is according to the Index 1 44 Integers.

Likewise the Number answering this Logarithme,

$\overline{2}$ 72028 is '052515
to — $\overline{3}$ 72028 is '0052515
to — 1 72028 is '52515

Thus much concerning finding the *Logarithmes* of
Numbers.

4. To adde two Logarithmes together, or substract
ne from another, is no difficulty if the Indices be not
egative, as for Example.

Add.	2.57832	Sub.	2.57832
	1.62517		1.62517
	4.20349		0.95315

But if the Indices be negative, worke them as was
ught before, changing the nature of the lower in
ubstraction; but remember that if in Addition you
rry any tenths they are affirmative, but in Substra-
ion if you borrow one, account the higher one lesse
value by Negation, as if it be 2 account it 1. if $\overline{2}$.
count it $\overline{3}$.

Addition.

$\bar{2}$.05782	$\bar{1}$.39794	2.15836
3.58321	$\bar{1}$.87506	$\bar{1}$.87506
1.64103	1.27300.	2.03342

$\bar{1}$ 87506	$\bar{2}$.23724
2 69897	$\bar{1}$.87506
2 57403	2.69897
	0.81127

Substraction.

2.03342	$\bar{1}$.87506	$\bar{1}$.87506	1.235
$\bar{1}$.87506	2 03342	2.57403	3.57
2.15836	3.84164	1.30103	3.663

5. To finde the Logarithme of a Fraction, subst[ract] the *Log.* of the Denominator, from the *Log.* of t[he] Numerator.

3	0.47712	3	0.47712	31	1.49136
4	0 60206	341	2.53275	572	2.75663
.75	1 87506		3 94537		2.73473

6. To multiply a Logarithme, whose Index is [or]dinary; but if the Index be Negative, observe tha[t] multiplying the next Figure to the Index, the tens

e borne in minde are affirmative, and so many are to e deducted out of the Product of the Negative In-ices.

1.54321	$\overline{2}$.54321	$\overline{4}$.61432
3	3	4
4.62963	$\overline{5}$.62963	$\overline{14}$.45728

7. To divide a Logarithme that hath a Negative ndex, observe whether the Divisor will evenly di-de the Index, then is there no difficulty, as in these xamples.

) $\overline{4}$.32578	3) $\overline{9}$.58321	5) $\overline{15}$.32571
$\overline{2}$.16289.	$\overline{3}$.19440	$\overline{3}$.06514.

But if it doe not evenly divide the Index, adde to e Index so many Unites till it may be evenly divi-d, setting the Quotient downe for a new index, eping those Unites added in minde, augment them ten, which Product adde to the next Figure, as in e Examples.

$\overline{5}$.32141	3) $\overline{7}$.23215	2) $\overline{5}$.61228
$\overline{2}$.44047	$\overline{3}$.74405	$\overline{3}$.80614.

Sect. 2. The use of the Logarithmes.

1. Multiplication.

Add the Logarithmes of the Multiplicand and Multiplyer together, the summe is the Logarithme of the Product, the Index shewing its Nature.

Md.	3.42	2.53402	75'8	1.87967	3'42	0.534
Mr.	13	1.11394	92	1.96378	'13	$\bar{1}$.113
P.	4446.	3.64796	6973'6	3.84345	'4446	$\bar{1}$.64

'75	$\bar{1}$.87506	144.	2.15836
'65	$\bar{2}$.69897	'75	$\bar{1}$.87506
'0375.	$\bar{2}$.57403	108'00	2.03342

172'68	2.23724	'037.	$\bar{2}$.56820
'75	$\bar{1}$.87506	'008.	$\bar{3}$.90309
'05	$\bar{2}$.69897	'000296.	$\bar{4}$.47129
6'475.	0.81127		

To square, cube, square-square, &c. any Number, is to double, triple, quadraple, &c. its *Log.*

N. 32.	1.50515.	1.50514	1.5051
	1 sq. 2	Cub. 3	sq.sq.
	3.01030	4.51542	6.0205
square is.	1024.	Cub. 32766.	sq.sq. 10485

N '024.	$\bar{2}.38021$		$\bar{2}.38021$
	ſq. 2		Cub 3
'000576.	$\bar{4}.76042$	'0000138.	$\bar{5}.14063$

2. *Diviſion.*

Subſtract the *Log.* of the Diviſor from the *Log.* of the Dividend, the remaining *Log.* is the *Log.* of the Quotient.

Dd.	4446.	3.54796.	4'446	0.64796
Dr.	342	2.53402	3'42.	0.53402
Qt.	13.	1.11394	1'3.	0.11394

'0375	$\bar{2}.57403$	108	2.03342
'05	$\bar{2}.69897$	'75	$\bar{1}.87506$
'75	$\bar{1}.87506$	144'00.	2.15836

To extract the ſquare Cube, and ſquare-ſquare Root, &c. is to take halfe a third part, &c. of the *Log.*

N				
75832,	2)	4.87985	3)	4.87985
		2.43992.		1.62662.
ſquare Root	275'37		Cube Root. 42'327.	

'05213.	2)	$\bar{2}.71708$	3)	$\bar{2}.71708$
		$\bar{1}.35854$		$\bar{1}.57236$
ſq. Root is	'2283		Cube Root is. '36506.	

Sect. 3. *Of* Compound Usery, *or* Interest *upon* Interest.

Though the taking and bargaining after this manner of Interest be complained of by many who understand not what they speake, yet it is easie to make it appeare that it is far more reasonable in all *Bargains*, then that which is called *simple Interest*, for what is more unjust then after 8 *per cent.* for a yeare to take 4l. for the use of 100l. for $\frac{1}{2}$ a yeare, or 2l. for a quarter, when as that 4l. will give in halfe a yeare 3s. - 2d ob. and 2l. in 3 quarters will give two shillings five pence: But in Leases, Reversions, Annuities, &c. it is far more unreasonable, as divers Compounders at G. H. can by wofull experience witnesse, who compounding according to the Rules of simple Interest, have payd more for their Tenements, Annuities, Leases in Reversion, &c. then they have beene really worth.

1. The Interest in this Kingdome is rated after so much gaine in the 100l. for a yeare: Whatsoever therefore it is, whether 108. or 106. finde out its *Log.* the which if the payments be halfe yearely, or quarterly, or monethly, take its $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{12}$, accordingly; then will your Question fall upon some of these six fundamentall Theoremes invented and set downe by Mr. *Oughtred*, the practize whereof followeth.

1. To know what ought to be payd for any sum forborn

forborne for any number of yeares, *Interest* upon *Interest*, at any Rate propounded.

2. To know what any summe due after a certaine time is worth in ready money.

3. To know what ought to be payd for an *Annuity* discontinued for any time, at the end of that time.

4. To know what *Annuity* any summe that is due after a certaine time will buy.

5. To know what an Annuity due for many payments is worth in ready money.

6 To know what Annuity a present summe of money will buy for any time proposed.

2. For the more speed finde out the *Log.* of the Rate, which divide by 2. 4. 12. 36. for the parts of the yeare.

at 8 *per cent.*		at 6 *per cent.*	
1.08. Log.	0.0334237.	1.06. Log.	0 0253058
$\frac{1}{2}$ yeare.	0 0167118.	— — —	0.0126529
$\frac{1}{4}$ yeare.	0 0083559.	— — —	0.0063264
month.	0.0027853.	Month.	0.0021088
weeke.	0.0006427	Weeke.	0.0005272
day.	0.0000915.	Day.	0.0000753

Lastly finde the Interest, or resolve the Question by 1l. and adde the *Log.* of the Answer to the *Log.* of the summe propounded.

Question.

Question 1.

To know the Intereſt of any ſumme for any time required.

Multiply the Logarithme of the Rate by the years or time propounded.

1. *Examp.* To know the uſe of 25l. – 8s. for 7 yeares, at 8l. *per. cent.*

1.08 Log. of the Rate	——	0.03342
		7
The Log. of 1.7138	—— ——	0.23394
To which adde the Log of 25'41.	——	1.40483
		1.63877
The Log. of Principall and Uſe	—— —	43.529 viz. 43l. – 10s. – 7d.

2. What is due for 25 l. 8s. for three yeares and a quarter.

Log. of the Rate.	4)	0.03342
Log. of the quarterly Rate.		0.00835
the Quarters in 3 $\frac{1}{4}$		13
		0.02505
		00 0835
The Log of Principall and Uſe for 1l.	—— —	0.10855
To which adde the Log. of 25'4		1.40483
		1.51338

The Principall and Uſe is 32.632, that is 32l. 12s. 7d. 3f.

3. What

3. What is due for 13l. – 5s. for a Moneth.

Log. of the Rate divided by 12) ———	0.02342
	0.00278
Adde the Log. of 13l.25	1.12221
	1.12499

The answer is 13.'319.
that is 13l. – 6s. – 4d. – ob.

What will 17l. 15s. amount unto at the end of 3 yeares and a halfe, 2 moneths and 8 dayes, at the rate of 6 *per cent.*

Rate. 1'06. Log. 0.02530 by 3 is –	0.07590
½ yeare is ——— ———	0.01265
A moneth is 0.00211 by 2 is.	0.00422
A day is 0.00007 by 8 is.	0.00056
Interest for 1l. ———	0.09333
Log. of 17l.–15s. —	1.24919
22'005	1.34252

The answer is 22l. – 0s. – 1d.

The same worke may be observed for any other broken or uneaven time.

Question 2.

To know what any summe due at a time to come is worth in ready money.

Finde by the last what 1l. would come to in the time

time proposed, which substract out of 0.00000, the Log. of 1l. to this Log. adde the Log of the summe: You have the resolution.

1. What is 30l. to be payd at 7 yeares end, worth in ready money.

Log. of the Rate 1.08.	0.03342
	7
	0.23394
	0.00000
	1 76606
Log. of 30l. ———— ————	1.47712
An. 17l.'506 that is 17l.10s.1d.	1.24318

2. What is 40l. due at 2 yeares hence, worth in ready money after 6l. *per cent.*

Log. of the Rate ——	0.02530
	2
	0.05061
	0.00
	1 94939
Log of 40l. ————	1.60207
An. 35'6 or 35l. – 12s. ——	1.55146

Question 3.

What any *Annuity* is worth to be payd at the end of the Tearme.

For

For resolving of all *Questions* about *Annuities*, you must finde out the Log. of the Rate, and its Number: as also the Log. of the Principall and Use, and its Number by the 1. Q. From both which Numbers substract an Integer, and then find the Correspondent Log. of the remaining Numbers.

Lastly, substract the Log of the Rate lesse by 1. from the Log. of the Principall and Use lesse by 1. the remaining Log. shewes the value of the Annuity for 1l. to which adde the Log. of the summe for the Resolution.

1. There is an yearely Annuity of 15l. – 11s. – forborne for 7 yeares, what is to be payd at the end after the rate of 8. *per centum*?

1.08 Log. of the Rate.	0.03342	'08	$\bar{2}$ 90309
	7		
Principall & use 1.7138.	0.23384	'7138	$\bar{1}$ 85357
1.08 lesse by 1. is '08		val. of 1l. 8.9224.	0.95048
1.7138 lesse by 1. is '7138		value of 15.l 11s.	1.19173
		138'74 ——	2.14221

The summe to be payd at the end is 138 l. – 14s. – 10d.

2. There is a Quarterly Annuity of 15l. – 11s. – forborne for three yeares and a quarter.

What is to be payd at the end? The Rate after 6 *per cent.*

1.06

1.06. the Rate 0.02530.
Quarterly Rate (4) 0.00632. 1.0147.
Quarters. 13
0.01896
0.0632
Principall and Use. 1.2083.
0.08216

1'2083. lesse by 1. is '2083. Log. $\bar{1}$.31868.
1'0147 lesse by 1 is '0147 Log. $\bar{2}$.16731
Value for 1l. ——— ——— 1.15137
Log. of 15l. - 11s. ——— ——— 1.19173
220'34 2.34310
To be payd at the end 220l. - 6s. - 10d.

Question 4.

What Annuity any summe due after a certaine time will buy after a rate for any time proposed?

The operation differs from the last onely in this, that the Log. of the Principall and Use lesse by 1. is to be substracted from the Log. of the rate lesse by 1.

352l. - 10s. is due after 7 yeares; what yearly Annuity will it buy for that time? the rate 8 *per cent.*

Log. of the Rate 1'08. 0.03342. '08. $\bar{2}$.90309
7
Principal & Use 1.7138 0.23384 '7138 $\bar{1}$.85357
Value of 1l. ——— ——— ——— ——— 1.04952
Log. of 352l. - 10s. ——— ——— 2.54716
Log. of ——— 39'510 ——— ——— 1.59668
That is 39l. 10s. 3d. yearly.

Question

Question 5.

To know the value of an Annuity in ready money, e for many payments at a Rate given.

Obtaine the Log. of the Principall and Use lesse by out of which substract the Logarithmes of the ate lesse by 1. and of the Principall and Use added gether.

1. What is 60l. a yeare to continue 30 years worth ready money, at 8 *per cent*? *An.* 675l. 9s. 4d.

98 Log. ———	0.03342	
	30	
10.06	1.00260	
'08	$\bar{2}$.90309	Log. of Rate lesse 1.
	$\bar{1}$ 90569	
inc. & Use lesse 1.	0.95713	
	1.05144	the value of 1l.
	1.77815	Log of 60
	2 82959	Log of 675.46

Question 6.

To know what Annuity any present summe will y for a certaine time, after any Rate propounded.

The Rule differs little from the former, onely sub- ct the Log. of the Principle and Use lesse 1. from Log. of the rate lesse 1. and the Principall and Use ded together.

1. What

1. What quarterly Annuity for 7 yeares will 13 buy after the Rate of 8 *per cent*? An. 6l. – 3s.

1.08	.0.03342.	
4)	.0.00835	Rate. 1 0194.
	28	Quarters in 7 yeares
	6680	
	1.670	
	0.23380	Principall and Use. 1.7132
	$\bar{2}$.28780	Rate lesse 1. is 0194
	$\bar{2}$.52160	
	1.85327	Principal and Use lesse 1. is 7132
	2.66833	value for 1l.
	2.12057	Log. of 13.2l.
	0.78890	Log. of 6.1504

The like may be done for halfe yearely or monethly payments or pensions, and at what rate is desired.

The same practice may be made by the Log. in the other parts of Arithmetick.

Ex. If 17l. 3 ounces of silver cost 32l. – 11s. – 8d. what will 132l. and 2 ounces cost? *An.*

32l. – 11s. – 8d.	32.583.	.Log.	1.51299
132. 2. oz.	132.16	.Log.	2.12110
		Summe —	3.63409
17l. 3.	17. 25.	Log. —	1.23678
Log. of 249'66			2.39731

The answer is 249l. – 13s. – 2d.

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other Literature,

The Author [illegible] this Second Book
[illegible], and Dedicateth the
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Shave had Company

Part, 2.

Chap. I.

Notatio Of Notation.

1. ARithmeticke in *Species* teacheth from a thing *unknowne* and *suppositious*, to finde truely that which is sought, and differeth from *Arithmeticke* in *Numbers*; herein in that the quantities to be measured are expressed by certaine Figures or characters proper to the Art, in this by certaine Alphabeticall Characters, the things themselves (as it were in their *Species*) are signified and altogether handled; from the which it draweth its name.

2. And heerein wee consider, 1 the *Notation*, 2 the *Numeration*, 3. the *Equation* (which wee consider also in three parts: 1 The *Composition* of the Question, or Intention of the Equation. 2 the Proprieties and *Reduction*) 3, the *Resolution* of the Æquation.

 3. The

3. The Notes or Characters used in this Art (as was sayd before) are commonly the Letters of the Alphabets A. B. C. &c. a. b. c. and α. β. γ &c. and therein also are used many other Symboles or Characters for taking away the Multiplicity of words, as = for equall, + for more or the Figure of Addition, — for lesse the signe of Substraction × for in, or by, or multiplyed, √ for a root, √: for a root universall ⊏ for greater or majority, ⊐ for lesse or minority, :: for proportion ⫶ the greater proportion, ⫶ the lesser ∺ continuall, &c.

4. In this Arithmeticke, the species of the same kinde, which by the power of Multiplication ascend, are called *Species ascending*, and in the vulgar way, Numbers Cossicke.

Indices	1	2 .	3 .	4 .	5 .	6 .	
Root. 1.	2.	4 .	8 .	16 .	32 .	64 .	
	A.	Aq.	Ac.	Aqq.	Aqc.	Acc.	
	7 .	8 .	9 .	&c.			
	128 .	256 .	582 .	&c.			
	Aqqc.	Aqcc.	Accc.	&c.			

The lowest row of species are the powers of A the roote ascending, as Aq signifieth that A is squared or A square, Ac, A Cube, Aqq A square-square, Aqc A square = Cube, Acc A Cube-Cube. Aqqc A square-square-Cube, &c.

The middle Figures signifie the powers in Numbers

if 2 be put for the roote then, 4 is the square of 2. 8 the c of 2. 16 the qq of 2. 32 the qc of 2, &c. and these proceede from 2 in a continuall proportion.

The uppermost or highest Numbers are the *Indices* or *Exponents* of the Powers, shewing the Powers themselves, that is how far distant from the roote, or what proportionall it is in the progresse, and these increase in an *Arithmeticall* Proportion.

5. For how many unites there are in the Exponent; so many are the powers or proportionalls from unity, as if the Exponent be 5, the power is a qc, 1. 2. 3. 4. 5.
or 1. a. aq. ac. aqq. aqc.

6. Every power is made of the Multiplication of the roote so many times into it selfe, as there are unites in the exponent; as if the Index be 4 then. $A \times A = Aq \times A = Ac \times A = Aqq$ the power under the Index 4 or $A \times A \times A \times A = AqAq = Aqq$: but more plainely of this in Multiplication.

7. The summe of any two Exponents, doe intimate the Plane made by the Multiplication of the Powers shewed by them.

$2 + 5 = 7$		$3 + 4 = 7$	
$4 \times 32 = 128$	Likewise	$8 \times 16 = 128$	
$Aq \times Aqc = Aqqc.$		$Ac \times Aqq = Aqqc.$	

8. The difference of any two Exponents, doe in-

timate the Plane made by Division of the greater power by the lesse shewed by them.

$$\begin{array}{lll} & 5 - 2 = 3 & \qquad 4 - 3 = 1 \\ \text{As } 32: & 5)\ 32\ (8 \quad \text{Likewise} & 16.\ \ 8)\ 16\ (2 \\ & Aqc.\ Aq)\ Aqc\ (Ac. & Aqq.\ Ac)\ Aqq(A. \end{array}$$

9. Therefore if any Exponent be given, the Cossicall Power may thereby be found out thus; the Exponent is either prime or compound: If prime seek it out in the following lines, the Power under expresseth it.

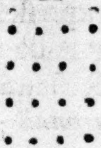
$$\text{If it be}\begin{cases} 2. & 3. & 5. & 7. & 11. & 13. & \&c. \\ Aq. & Ac. & Aqc. & Aqqc. & Aqccc. & Aqqccc. & \&c. \end{cases}$$

compound, multiply the power answering the greater number so many times as there are unites in the lesser, whereof the index was compounded as to the Index of 10 made of $5 * 2$, that is $Aqc * Aqc =$ Aqqcc, and so of any other.

10. And therefore contrarily the Exponent or Index of any Power may be gotten after the same meanes.

11. By sect. 5. the *Composition* of the Powers and by that also the definition of the *Roote* of any Cossicke Number appeares, which is the side of first *Species* whereof the Power was made: As the roote square of Aq or $\surd$ Aq is A, the $\surd c$ of Ac is A. the $\surd qq$ of Aqq is A, &c.

A



4 * 4 = 16 the roote 4.

A. A * A = Aq the roote A.

5 * 5 * 5 = 125 √5.

A. A * A * A = Ac √A.

12. But if the Roote be unexpressible in Numbers
r Quantity, as if the *Ratio* betwixt 1, and the √
nnot be expressed, the roote thereof is called *Sur-*
, because properly it cannot be expressed; and it
noted as was sayd before by √. As suppose B to
gnifie 10, then the roote of B is no way to be ex-
essed then thus. √B. so the Cube, sqq. sqc. are thus
pressed √c B. √qq. B. √qc B, &c. Sometimes
o it is desired to expresse the rootes or powers of a
rde roote, as the square roote of √q 5. the Cube
ote of the √qq 74, &c. to doe which, you must
ltiply the Indices of the two powers named for a
w Index; as √q of √q 5 is √qq 5. *viz.* 2 * 2 = 4.
√c of √qq 74 = √cccc 74, *viz.* 3 * 4 = 12, and
of any other.

13. Sometimes the roote of many species is ex-

pressed after the same manner; but then after the last species we joyne a *Colon* thus, √B+D: √c C + E-D:

14. If there be more simple species expressed besides the √, then is such a species if the signe be affirmative called a *Binomiall*, if Negative a Residuall, if many then is such a *Trinomiall* or *Polynomiall*; and these are accounted Compound Surds.

15. But if the compounded species be such a *Binomiall Trinomiall*, &c. and yet the roote of the whole be to be expressed, then is such a species called an universall Surd, and may be noted thus √B+√c or thus √ : B+√c.

16. Sometimes it falls out that wee must expresse the roote of a Cossicke *species*, as the Cube roote of Aqc. to be thus expressed √cAqc. and so of any others.

17. Thus are all rooted Magnitudes explained; now he that desires to enjoy the great benefit hereof must accustome himselfe to expresse all quantities whether of Number or Magnitude by their species which if they consist of one letter are called Simple, if of many Binomicall or Polynomicall: The signification whereof is first to be learned: As if A and E signifie two Numbers; let Z signifie the summe of these Numbers, X the difference, Æ the rectangle or Multiplication, $\frac{R}{S}$ the proportion, Z the summe of the squares, X the difference of the squares, Z the summe of the Cubes, and X the difference of the Cubes of them.

And if you appoint A to be 3, and E to be 2; then vill Z be 5. X 1. Æ 6. S 4. Z 13. X 5. Z 35. X 19.

Now if you would expreſſe AqE, in words it ſig-ifieth that (if you appoint A for the greater, and E or the leſſer Number) the greater Number ſquared is be multiplyed by the leſſe in Numbers $9 \times 2 = 18$.

ZA-Aq. in words, that the ſumme of the two umbers multiplyed by the greater, is to be leſſened y the ſquare of the greater, which in Numbers is $3 - 9 = 6$.

Zq-2ZE+Eq. in words, requireth that you ſub-act the ſumme of the two Numbers multiplyed by e leſſer Number doubled, from the ſumme of the vo Numbers ſquared, added to the ſquare of the ſſ r, in Numbers $25 - 20 + 4 = 9$.

$\frac{SqAq}{Rq.}$ In words ſignifieth that the ſquare of the leſſer proportionall is to be multiplyed by the ſquare of the greater number, and that e product is to be divided by the ſquare of the grea-er proportionall, in numbers $\frac{16 \times 9}{36}$ — $\frac{144}{36} = 4$.

$\sqrt{Z-Aq}$. Signifieth that the ſquare of the grea-ter is to be ſubſtracted from the ſumme f the ſquares of the two Numbers, and that the uare roote thereof is to be taken, in Numbers $\sqrt{13-9} = \sqrt{4} = 2$.

$\frac{\sqrt{ZRq-RqAq}}{Aq.}$ Signifieth that you muſt take the ſquare roote of the Quoti-

ent

ent of these two Quantities, *viz.* the summe of the squares multiplyed by the greater Prop. and made lesse by the same in the square of the greater Number, the Remainer is the Dividend, and the square of the greater the Divisor in Numbers.

$$\frac{468-324}{9} = \frac{144}{9} = \sqrt{\frac{144}{9}} = \frac{12}{3} = 4.$$

$\frac{Z}{2} + \sqrt{\frac{Zq-4P}{4}}:$ Signifieth that you must adde the halfe of the summe of the two Numbers, unto the square roote of the summe squared, made lesse by foure times the Rectangle, and divided by 4, in Numbers.

$$\frac{5}{2} + \sqrt{\frac{25-24}{4}} = 2\tfrac{1}{2} + \sqrt{\tfrac{1}{4}} = \tfrac{1}{2} = 3.$$

$\sqrt{:}\,\frac{Z}{2} - \sqrt{\frac{Zq-4Pq}{4}}$ Signifieth that you must take the universall square roote of halfe the summe of the squares made lesse by the square root of the sum of the squares squared, lessened by 4 Rectangles of the Numbers squared and divided by 4, in Numbers.

$$\sqrt{:}\,\tfrac{13}{2} - \sqrt{\frac{169-144}{4}} : = \sqrt{:}\,6\tfrac{1}{2} - \tfrac{25}{4} = \sqrt{:}\,6\tfrac{1}{2} - 2\tfrac{1}{2} = \sqrt{4:} = 2$$

But if the signe of equality come in, then in words it will not much differ; as for example.

$E = \frac{Zq - X}{2Z}$ Signifieth that the lesser Number is equall to the Quotient of the summe squared, lessened by the difference of the squares, and the Remaine divided by the summe of the Numbers doubled, in Numbers.

$$2 = \frac{25 - 5}{10} = \frac{20}{10} = 2.$$

$2Aq - 2XA = Z - Xq$. Signifieth that the square of the greater doubled, made lesse by the greater multiplyed by the difference doubled, is equal to the summe of the squares, made lesse by the difference squared; and so of any other, and after the same manner you may expresse any of the forementioned words in Characters, as to expresse thus much, that the greater Number is equall to the difference of the squares, made lesse by the difference of the Numbers squard & divided by the same difference doubled it would be $A = \frac{Z + Xq}{2X}$.

Now for that the consideration and varieties of operations of two Numbers are of great use in the practise of this kinde of Arithmeticke; many questions in *Diaphantus*. *Vietta*, *Ghetaldus*, &c. being of some of their parts, I have prefixed a Table at the end of Chap. X. for the present finding the value of any Æquation in Numbers for triall of the worke to a new

new beginner, putting A $=$ 3, and E $=$ 2.

18. But I must heere give notice to the Reader, that although I in this practise appoint these letters to signifie the Numbers proposed, yet may those letters signifie any other Figures or Quantities, according to the will of the *Arithmetician*; as in Mr. *Oughtreds* Cla. Chap. XIX. Prob. VI. where he appoints for the Solution of Problemes in an Arithmeticall progression, α for the first tearme, ω the last, T the Number of Tearmes, X the common difference, Z the summe of all the Tearmes, and therefore his 1 Prop. $T\omega - T\alpha = 2Z$ to be read thus: If you adde the Product of the Number of the Tearmes multiplyed by the last Tearme, to the Product of the Number of the Tearmes multiplyed by the first, the summe of these two Products is equall to the summe of the Tearmes doubled.

19. For distinction sake, note alwayes the given Quantities or Numbers with Consonants, and those which are sought with Vowells, lest you make a confusion in your worke; and note as was taught in the former Introduction, that if the signes − be not prefixed before a species, the signe + is there understood.

20. All Magnitudes that are under the Power proposed, are called *Parodicall to the Power*: as for example, the *Parodicall degrees* to Aqq are the root A the square Aq and the Cube Ac.

21. And if the *Parodicall* degree have a known Magnitude joyned with it, it is called the *Coefficient*

with

ave a

s in

tions

to an

Root,

ion ,

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able.

ation

- *Arithmetick* in *Species* is either
  - *Simple*, consisting in
    - *Notation* by Alphabeticall Symbolls and Characters;
    - *Numeration* which is
      - Addition, Substraction, Multiplication, Division, and that in
        - Rationall species,
          - Either surde or co[m]pound, which [is] Binomial and [Po]lynomiall,
        - Surde.
          - And these either si[m]ple, or compou[nd] or Universall[s]
  - *Comparative* in the great Rule of
    - *Algebra*, or *Æquation*, which we consider in the
      - *Invention* of the Æquation,
        - The worke preceding it
      - *Reduction* of the Æquation.
        - The worke concomita[nt] with the Æquation.
      - *Resolution* of the Quation.
        - The worke following i[t] or subsequent, which [is] done either by Division or extraction of the root[s]

Place this Table in the second Alphabet Fol. 11, at this Marke. ☞

Aq - AB. B is the Coefficient, and A joyned with is the Parodicall degree under Aq.

22. The word Roote may likewise heere have a ouble signification, for whilest a Question is in orking, wee call A the *suppositions* or quæsitions oote: But when the Question is brought to an quation, then you may call it the *Eductitions Root*, the Roote to be drawne out.

23. Thus have wee gone through Notation, ith a desire to make the beginning plaine, whereof, of that which shall follow, take this little Table.

☞

Now come wee to proceede to the rest in order, and first of the parts of Numeration, *viz.* Addition, Substraction, Multiplication, and Division of Rationall Species.

Chap. II.

Addition of Rationall Species both simple and compound.

1. ADdition joyneth together the Quantities given, keeping the signes, as,

Quantities to be added.	3A	A	6B	9E
	A	− A	− 7B	− 5E
The summe.	3A+A	A − A	6B − 7B	9E − 5E
That is.	4A	0	— 1B	4E

A	A	Aq
E	− 5E	A+Ac
A+E	A − 5E	Ac+Aq+A.
A+E	A − 5E	

Quantities to be added.	A+B	A+D	A − B
	A+B	A−5D	−A − B +C
The summe.	A+B+A+B	A+D+A − 5D	A−B −A−B+C
That is.	2A+2B	2A — 4D	C − 2B.

Quan-

Quantities to be added.	$5A - 10B + 7D$ $-2B - 5C$	$A + B - C$ $A - B + 2C$
The summe.	$5A - 10B + 7D 2B - 5C$	$A + B - C + A - B + 2C$
That is.	$5A - 12B + 7D - 5C$	$2A + C$

2. Heere it is to be noted, that if in any Quantity onsisting of many species or parts annexed, with he signes + or − there be two species noted with the me letter, that then they must be made up into one pecies, and noted with one letter, by prefixing their ommon signe before their summe, if their signes be oth alike, or by prefixing before their difference the gne of that letter wherein the excesse did lye, if their gnes be unlike.

As in the first example, $3A + A$. Heere these two pecies are made one, *viz.* $4A$ as the summe of both, ecause the signes are both alike, *viz.* both +; for + is nderstood before $3A$ though it be not expressed.

In the second example $A - A$, the signes are divers unlike, for + is understood before the first A: and erefore their difference is nothing, and to no more ey come to, the latter taking away the former, and the summe is noted with a Cypher 0.

In the third Example $6B - 7B$, the signes are unke, + being understood before $6B$, and therefore they e made up into $-B$, which is their difference of $7B$ d $6B$, having the signe − prefixed before it, which

is

is the ſigne where the exceſſe lay, *viz.* upon 7B.

In the fourth the ſumme 4E being the differen
of 9E and 5E, and the ſigne + becauſe the exceſſe w
on +9E.

In the fift becauſe the ſpecies are unlike, they a
joyned with the ſigne of Addition +, *viz.* A+E, a
ſo in the ſixt A — 5E.

In the eighth the ſignes are made up 2A+2B.

In the ninth I make up +D - 5D into - 4D, a
A+A into 2A, which is 2A - 4D.

In the tenth finding +A A I cancell them bo
and make -B - B to be - 2B, and then the ſumm
C - 2B. and ſo of the reſt.

CHAP. III.

Of Subſtraction.

1. S*Ubſtraction* in ſpecies j
neth together both
Quantities given, ch
ging all the ſignes of
Quantities to be ſubſt
cted, *viz.* making in
lower Number the +
be — and the ſigne —
be +.

om —	A	5A	5A	2A
ſtract —	A	2A	-A	5A
re remaineth	A - A	5A - 2A	5A + 2A	2A — 5A
t is —	0	3A.	7A.	-3A

A	A	Aq
E	B - C	A
A - E	A - B + C	Aq - A

m —	A	3A + 5B
ſtract —	B + C	4A + B
re remaineth	A - B - C	3A + 5B — 4A — B.
is —		4B — A.

m —	5Aq — 2BA — CD
ſtract —	CD — 3Dq — Aq
re remaineth	5Aq — 2BA - CD - CD - 3Dq + 4Aq
t is —	5Aq — 2BA — 2CD + 3Dq.

After you have ſet downe your two quantities, ing all the ſignes of the lower, you muſt then ve to joyne together like ſpecies, as in Addi-

s in the ſecond Example, make + 5A, - 2A by ging the ſigne, I joyne them together into 3A, the

the ſigne being + and belonging to 5 A, where the exceſſe did lye.

3. But you muſt note that both in this as in Addition, that if the ſpecies have Coſſicke ſignes joyned to them, they cannot bee added together but with another letter that hath the like Coſſicke ſigne.

As in the ſeventh Example of either, Aq and A cannot be added, unleſſe by + and - becauſe they are not of the ſame power, the one being the ſide, and the other a ſquare number; but if they had been Aq + Aq then they had amounted to 2 Aq.

4. In the ſixth Example where you change the ſigne of - C, and make it + C, it is by way of Compenſation; for if you take - B out of A, you take too much by the quantity C, which therefore is recompenced againe by making C affirmative, or adding it to A: in numbers it is plaine let A = 12 B = 8 C = 4.

$$\begin{array}{lr} & 12 = 12 \\ \text{Let them ſtand thus.} & 8 - 4 = 4 \\ \hline & 12 - 8 + 4 = 8 \end{array}$$

5. As in Numbers, ſo in *Species*, if you adde the Remainder, and the ſpecies to be ſubſtracted, they will make up the Quantity, from which the Subſtraction was made: As in the third Ex. 7A - 2A = 5A

Chap. IIII.

Of Multiplication.

1. M*ultiplication in Species* connecteth both the Quantities propounded without any note at all; but if the Quantity be noted with two letters, then must you use the signe *.

2. Jn multiplication, if the signes of the Quanti-ties to be multiplyed together be alike, that is both + or both — then the signe of the Product is affirma-tive or +, but if they be unlike, then it is negative or —.

3. If Numbers be annexed to the species that are to be multiplyed, then you must multiply those Num-bers together, and joyne the Product to the Product of the species multiplyed with the correspondent signe.

Examples of Multiplication.

*	Aq	Ac	Aqq	Aqc		Aqc	5+	Aqc	5
	A	A	A	A	&c.	Aq	2	Aqq	+4
q	Ac	Aqq	Aqc	Acc.		Aqqc	7	Accc.	9

These Examples are in species of one kinde, and therefore in the 5 first examples A*A = Aq. Aq*A = A*A*A = Ac, and so of the rest, in the sixth and

seaventh examples to multiply Aqc of the 5 power by Aq of the 2, adde the Indices $5+2 =$ seaventh power, that is Aqqc as was taught Chap. 1. Sect. 6, &c.

But if your species be Binomiall as of $A+E$, then the powers arise as followeth, *viz.*

$A+E$
$\underline{A+E}$ Lat.
$Aq+AE$
$\underline{\quad +AE+Eq}$
$Aq+2AE+Eq = Q{:}\ A+E.$
$\underline{\qquad A+E}$
$Ac+2AqE+AEq$
$\underline{\quad +AqE+2AEq+Ec}$
$Cc+3AqE+3AEq+Ec = C{:}\ A+E.$
$\underline{\qquad\qquad A+E}$
$Aqq+3AcE+3AqEq+AEc$
$\underline{\quad +AcE+3AqEq+3AEc+Eqq}$
$Aqq+4AcE+6AqEq+4AEc+Eqq{:}\quad QQ{:}\ A+E$
$A+E$, &c.

This is the Genesis of all *Binomiall* Powers, by meanes whereof not onely the *Genesis* but *Analysis* of all Powers or extraction of Rootes are demonstrated and taught, and that most excellent Posterior Table in Mr. *Oughtreds* Table is composed.

Md.

Md.	A	A+B	A — B
Mr.	B	B	BA
	BA	BA+Bq	BAq — BqA

$$\begin{array}{l} A+B+C \\ \quad D-C \\ \hline DA+BD+CD \\ \quad -CA-BC-Cq \\ \hline DA-BD-CA+CD-BC-Cq. \end{array}$$

$$\begin{array}{l} A+CD \\ BA-CD \\ \hline BAq+BCDA \\ \quad -DCA-CqDq. \end{array}$$

$$\begin{array}{l} AB-CB+AB \\ \quad BA-CB. \\ \hline BqAq-BqCA+BqAq \\ \quad -BqCA+CqBq-BqCA \\ \hline 2BqAq-3BqCA+CqBq. \end{array}$$

In the first example where A+E is multiplyed into it selfe; first I multiply A+E by A *facit* Aq+AE, then I multiply A+E by E *facit* AE+Eq which added makes Aq+2AE+Eq.

In the fourth Example where A - B×BA. here because A multiplies BA, it maketh BAq, and the signe + because they are affirmative, then because B multiplyeth BA it maketh BqA, and now the signe prefixed is negative, because the signe before was — and the signe before BA+ that is unlike: the which is observed in the Examples following.

Certaine Propositions for the use of Multiplication.

Prop. 1. If a Number suppose Z be cut into two unequall parts, that is $Z = A+E$, to finde unto what Plane the Multiplication of $A+E$ in it selfe shall be equall to:

$$\begin{array}{l} Z = A+E \\ Z = A+E \\ \hline Zq = Aq + 2AE + Eq \end{array}$$

By which I finde that the square of the whole line, is equall to the squares of either Segment added to the double Rectangle of the unequal parts, which is the 4 e. 2.

Prop. 2. If a Number be divided into two unequall parts; that is, $Z = A+E$, to finde out unto what Planes (having relation thereto) the square of the difference of these parts shall be equall.

$$\begin{array}{l} A - E \\ A - E \\ \hline Aq - AE \\ \quad - AE + Eq \\ \hline Aq - 2AE + Eq. \end{array}$$

Wherefore I conclude that the square of the difference of two Numbers is equall to the squares of the parts made lesse by the doubled Rectangle of the parts.

Prop. 3. If a Number be divided into two unequall parts, *viz.* $Z = A+E$. To finde out unto what Planes, the Rectangle of the whole line, *viz.* $A+E$, and the difference of the parts, *viz.* $A - E$ is equall

A+E

$$\begin{array}{l} A+E \\ A-E \\ \hline Aq+AE \\ -AE-Eq \\ \hline Aq-Eq. \end{array}$$

I finde by multiplying A+E by A-E. That the Rectangle made of the whole Number, and of the difference of the parts is equall to the difference of the squares of those parts.

Prop. 4. To finde unto what Plane having relation to the former parts, the square of the whole Number made lesse by the square of the difference of the parts, is equall unto.

By the first *Prop.* the square of the whole Number = Aq+2AE+Eq.

By the second *Prop.* the square of the difference of the parts = Aq-2AE+Eq.

The lower being substracted from the higher reseth 4AE.

Therefore I conclude that the square of the whole Number made lesse by the square of the difference of the parts, is equall to foure Rectangles made of the parts.

Prop. 5. To finde unto what Plane the addition of the square of the whole line, and the difference of the parts will be equall unto.

The square of the whole Number	Aq+2AE+Eq
The square of the difference —	Aq-2AE+Eq
The Addition will be ———	2Aq+2Eq.

Wherefore I conclude that the square of the whole

Number, added to the square of the difference of the Parts is double the square of the Parts added together.

4. All the Parts of the Power of any Binomiall simply taken without unity, are in continuall Proportion.

$$Q: A+E = Aq + AE + Eq \because$$

$$\text{for } \frac{AqEq}{Aq} = Eq \text{ the third Prop.}$$

$$C: A+E = Ac.\ AqE.\ AEq.\ Ec \because$$

$$\text{for } Ac.\ AqE.\ \frac{AqqEq}{Ac}.\ \frac{AqEqq}{AqE} \because$$

$$QQ: A+E = Aqq.\ AcE.\ AqEq.\ AEc.\ Eqq \because$$

$$\text{for } Aqq.\ AcE.\ \frac{AccEq}{Aqq}.\ \frac{AqqEqq}{AcE}.\ \frac{AqEcc}{AqEq} \because$$

5. Out of the consideration of Chap. XII. Sect. V. The powers of any Binomiall are suddainely made, by setting downe all the Parodicall degrees under the highest Potestas to the roote both wayes, and joyning them together contrarily, and at the last prefixing the unity as in this little Table. which are also made and continued by Arithmeticall Progression on either side in forme of a Triangle, and adding up every two Numbers above for middle intermediate under, as you may see in the Example.

			A			
			2			Aq
		3		3		Ac
	4		6		4	Aqq
	5	10		10	5	Aqc
6	15		20	15	6	Acc
7	21	35		35	21 7	Aqqc.

As it is desired to produce the QC of A+E. which is the fifth power; I first set downe all the Parodicall degrees of them both to the Roote, then I couple them contrarily sparing the highest Potestas of either, and then prefix the right Numbers to them as in the Table, and therefore conclude that Aqc+5AqqE+10AcEq+10AqEc+6AEqq+Eqc is the 5 power or QC. of A+E.

			5
Aqc	Eqc	Aqc	Aqc
Aqq	Eqq	AqqE	5AqqE
Ac	Ec	AcEq	10AcEq
Aq	Eq	AqEc	10AqEc
A	E.	AEqq	5AEqq
		Eqc	Eqc
			5

6. If any Power be to be multiplyed by another species, if that species be Negative, then it makes all the other power if before affirmative to be negative.

A+E by A+E by −B = −BAq − 2BAE − BEq.

Chap. V.

Of Division.

1. I F the Dividend be made or compounded of the Divisor, as the one Factor (which is easily seen by marking well the order of Multiplication) then the other Factor is the Quotient, (by the eight Section of Division in the former part.)

2. But if the Dividend be not so made (as commonly it falls out in this kinde of Arithmeticke) then you must set the Dividend over the Divisor in forme of a Fraction with a little line betwixt them; (by the last part of the second Section of the sayd Chapter of Division.)

3. If you observe the Proofe of Division, which is, that the Product of the Quotient in the Divisor, be equall to the Dividend; it will further you to this kinde of Division, by taking such a Number of your Quotient, as being multiplyed by the the Divisor will be equall to the Dividend.

Examples of Division in Species.

Dividend.

Divisor BA (A Quotient. 3 Aq) 6 Ac (2A. BA-BE (B. B) A-E)

Aqq) Aqqc (Ac. 7-4=3 Aqc) Aqcc (Ac. 8-5=3. Aq) Acc (Aqq. 6-2=4.

BA-BE (A-E. CA+A (C+1. A) BA-BE-CA+CE (A-E B-C)

$\frac{B}{A}$ (B | B+C (B+C A-E) (A-E | -BC ($-\frac{BC}{E}$ E) | -BC ($\frac{BE}{A}$ -A.)

These Examples need no explanation, only observe that like signes make + and unlike − as in Multiplication.

Chap. VI.

Of the foure Parts of Numeration of Fractions in Species.

It is very requisite for him who desires to be a perfect *Arithmetician* to be very ready in the worke of Fractions, and therefore to be cunning in the foure first

first Section of the XVI. Chap. of the first part; for whosoever shall understand them, will easily understand the worke of species following it, having a dependance thereon, and according to the parts of those fractions expressed.

Sect. 1. *Of the greatest common measure.*

5.6. $\frac{BA}{BC}$ the greatest common measure is B) $\frac{BA}{BC}$

$\frac{BCD}{BDA}$ the greatest common measure is BD) $\frac{BCD}{BDA}$

Sect. 2. *Of the Reduction of Fractions.*

6. Let B be reduced to an improper Fraction whose Denomination is A.

It will be thus $\frac{\frac{B}{A}}{BA}$ $\frac{BA}{B}$ the Fraction.

Againe let B+C be reduced to an improper Fraction whose Denominator is A.

It will be thus $\frac{\frac{B+C}{A}}{BA+CA.}$ $\frac{BA+CA}{A}$ the Fraction.

7.

7. Let $B+\frac{D}{A}$ be reduced to an improper Fraction:

It will be thus $\frac{\frac{B}{A}}{BA}$ $\frac{BA+D}{A}$ the improper Fraction.

Let $B-C\,\frac{R+D}{S}$ be reduced to an improper Fraction:

It would be thus $\frac{\frac{B-C}{S}}{BS-CS}$ $\frac{BS-CS+R+D}{S}$ the improper Fraction.

8. On the contrary; let the improper Fraction $\frac{BA}{A}$ be reduced to the Integer:

It will be thus $A)\,BA\,(B$ the Integer.

So $\frac{BA+CA}{A}$ $A)\,BA+CA\,(B+C$ the Integer.

likewise $\frac{BS-CS+R+D}{S}$ $S)\,BS-CS+R+D\,(B-C\frac{R+D}{S}$.

9. Fractions of unlike Denomination ought to be reduced keeping still the same value;

As $\frac{B}{CA}$ $\frac{R}{DA}$ reduced would be thus.

BD

BD RC

A) $\frac{B}{CA}$ $\frac{R}{DA}$ $\frac{BD}{DCA}$ $\frac{RC}{DCA}$

$\underbrace{D \quad C}_{DCA}$ The Fraction reduced.

BA DC

So $\frac{B}{C}$ $\frac{D}{A}$ reduced thus. 1) $\frac{B}{C}$ $\frac{D}{A}$ $\frac{BA}{CA}$ $\frac{DC}{CA}$

$\underbrace{A \quad C}_{CA}$ The Fraction reduced.

BAE CDE FDA

10. $\frac{B}{D}$ $\frac{C}{A}$ $\frac{F}{E}$ reduced $\frac{B}{D}$ $\frac{C}{A}$ $\frac{F}{E}$ / DAE

$\frac{BAE}{DAE}$ $\frac{CDE}{DAE}$ $\frac{FDA}{DAE}$ The Fractions reduced one Denomination.

…t. 3. *Of Addition and Substraction of Fractions.*

…f the fractions have not the same Denomination, …summe or Difference of the Numerators must be …ver the common Denominator.

…Sum or difference is $\frac{B+D}{C}$. $\frac{BD}{AC}$ $\frac{CD}{AC}$ Sum or difference is $\frac{BD+CD}{AC}$.

… the Fractions have not the same Base, then they … be reduced as in the last Section, and then added …bstracted.

$\frac{B}{CA}$ $\frac{R}{DA}$ Reduced, Added, or substracted, are $\frac{BD+RC}{DCA}$.

Sect. 4. *Of Multiplication of Fractions.*

$\frac{B}{D} \times \frac{C}{A}$ Prod. $\frac{BC}{DA}$. D$\times$ $\frac{CA}{F}$ or $\frac{D}{1} \times \frac{CA}{F}$,

B D

…od. $= \frac{DCA}{F}$ B $\frac{C}{A} \times \frac{C}{D}$ Prod. $\frac{BACq}{DA}$.

B D

Prod. $\frac{B}{C}$ A) $\frac{BA}{RE} \times \frac{DE}{CA}$ E) Product is $\frac{BD}{RC}$.

R C

Sect. 5.

Sect. 5. *Of Division of Fractions.*

$\frac{B}{1}\Big)\frac{Aq}{D}\Big(\frac{Aq}{BD}$. $\quad \frac{Bc}{C}\Big)\frac{Ec}{A}\Big(\frac{EcC}{BcA}$. $\quad A)\ \frac{\overset{B}{BA}}{S}\Big)\ \frac{\overset{R}{RA}}{C}\Big(\frac{SR}{BC}$

$\frac{A}{F}\Big)\ \frac{\overset{C}{CA}}{\underset{D}{DF}}\Big)\ \frac{\overset{B}{BA}}{\underset{R}{RF}}\Big(\frac{BD}{CR}$ $\qquad \frac{A}{R}\Big)\ \frac{\overset{B}{BA}}{\underset{C}{CR}}\Big)\ \frac{\overset{1}{A}}{\underset{1}{R}}\Big(\frac{C}{B}$

Sect. 6. *To adde or substract any part from a Fraction, or to finde a part named in 6 Propositions.*

Prop. 1. *To adde any part or parts of a Number given to the same Number.*

As for Example to this Fraction $\frac{3A}{5}$ adde his hal[fe] or $\frac{1}{2}$, adde 1 to $\frac{1}{2}$ it maketh $\frac{3}{2}$, then multip[ly] $\frac{3A}{5} \times \frac{3}{2} = \frac{9A}{10}$: for proofe if I suppose $A = 10$, th[en] $\frac{30}{5} = 6$, the halfe whereof $3+6 = 9 = \frac{9A}{10} = \frac{90}{10}$

Ex. 2. If to $\frac{3A+D}{E}$ I adde his $\frac{2}{5}$, I adde 1 to $\frac{2}{5}$ m[a]ke

eth $\frac{7}{5}$, by which I multiply $\frac{3A+D}{E} * \frac{7}{5} = \frac{21A+7D}{5E.}$

Ex. 3. Of a pure Fraction to $\frac{3}{5}$ adde his $\frac{1}{3}$, I multi-
ly $\frac{4}{3}$ by $\frac{3}{5} = \frac{4}{5}$.

Prop. 2. *To add the part or parts of a given Magnitude to any other of his parts.*

Ex. 1. If the given Quantity was 3B, to the $\frac{1}{3}$ hereof, I desire to adde the $\frac{2}{5}$. I adde the parts toge-
ber $\frac{1}{3}$ $\frac{2}{5}$ makes $\frac{11}{15}$, this Fraction I am to multiply in the
iven Number $\frac{3B}{1}$ $\frac{11}{15} = \frac{11B}{3}$

Ex. 2. If the Number $\frac{15A-12}{8}$ be proposed, and
be desired to adde its halfe to his third part, I adde
$+\frac{1}{3} = \frac{5}{6}$, then $\frac{5}{6} * \frac{15A-12}{8} = \frac{75A-60}{48.}$

Ex. 3. I would adde the $\frac{2}{5}$ of 7A to the $\frac{4}{7}$ of it: I
de $\frac{2}{5}$ to $\frac{4}{7} = \frac{34}{35}$, and $\frac{34}{35} * \frac{7A}{8} = \frac{238A}{35}$ for proofe I
ppose A 5, then 7A = 35. the $\frac{2}{5}$ is 14 the $\frac{4}{7}$ is 20 =
$= \frac{1190}{35} = 34.$

Prop.

Prop. 2. *To substract any part or parts from any Number whatsoever.*

Ex. 1. Let the given Quantity be $\frac{B+3A}{C}$ it is desired to substract the $\frac{1}{3}$, and the $\frac{1}{5}$ from it. I first adde together the parts $\frac{1}{5}$ $\frac{1}{3}$ $= \frac{8}{15}$, I substract that Fraction from $\frac{15}{15}$, or 1 that is $\frac{7}{15}$, which I multiply by the given Quantity $\frac{B+8A}{C}$ x $\frac{7}{15} = \frac{7B+56A}{15C}$.

Ex. 2. From $\frac{9}{20}$ take his $\frac{4}{5}$. First I subduct $\frac{4}{5}$ from 1 or $\frac{5}{5}$ rest $\frac{1}{5}$, by which I multiply $\frac{9}{20} = \frac{9}{100}$.

Prop. 4. *A Quantity being given to substract any part or parts from any other of his parts.*

Ex. 1. Let the given Quantity be $\frac{B+8A}{C}$ from the $\frac{1}{3}$ whereof it is desired to substract $\frac{1}{4}$: First I take $\frac{1}{4}$ from $\frac{1}{3}$ resteth $\frac{1}{12}$, by which I multiply the given Quantity *facit* $\frac{B+8A}{12C}$.

Ex. 2. Also take the $\frac{3}{4}$ of B from the $\frac{8}{9}$ of it: First I take $\frac{3}{4}$ from $\frac{8}{9}$ resteth $\frac{5}{36}$, I multiply that by the given Quantity, it makes $\frac{5B}{36}$ the proofe is plaine: for if you put $B = 36$. $\frac{8}{9}$ of $36 = 32$ $\frac{3}{4} = 27$ and $32 - 27 = 5 = \frac{5B}{36} = \frac{180}{36} = 5$.

Prop.

Prop. 5. *To finde the part or parts of any given Number.*

Ex. 1. Let the given quantity be $\frac{B+8A}{C}$ I would finde the $\frac{1}{3}$ and $\frac{1}{4}$ thereof, I adde together the $\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$ by this I multiply $\frac{B+8A}{C} \times \frac{7}{12} = \frac{7B+56A}{12C}$.

Ex. 2. I would finde the $\frac{3}{4}$ of B, I multiply $\frac{3}{4} \times \frac{B}{1} = \frac{3B}{4}$ if you suppose B $=$ 20 the $\frac{3}{4}$ of it is 15 $=$ $\frac{3B}{4} = \frac{60}{4} = 15$.

Prop. 6. *To finde out the principall Number, if any of his parts be given.*

Ex. 1. Suppose that $\frac{B+3A}{C}$ be the two thirds of any other Number, I would know that Number, I divide $\frac{B+8A}{C}$ by $\frac{2}{3}$ the Quotient is $\frac{3B+24A}{2C}$ which is the principall Number desired.

Ex. 2. It is desired that I should finde the principall, whereof B is the $\frac{4}{5}$, I divide $\frac{B}{1}$ by $\frac{4}{5} = \frac{5B}{4}$, now suppose you set B $=$ 8, then $\frac{40}{4} = 10$, for 8 is the $\frac{4}{5}$ of 10.

The six former Propositions if well practised, will be of great helpe in the solution of many intricate questi-

questions proposed in to be resolved, when as part or parts of any Quantity are to be added or substracted.

CHAP VII.

The parts of Numeration in simple Surds, and Surds Cossicke.

Sect. 1. *Of Addition and Substraction.*

1. AS in Fractions, so in all simple surds, you are to consider whe ther they be commensurable o no; if they be commensurable and when they are brought in to their least tea mes, they the be truely figurate.

2 They are added or substracted thus; for Additio square the sum of the Roots, and multiply that squa Quantity by the Common Divisor, prefixing befo the last Product the first Roote, in Substraction squa the difference, and worke as before.

$\sqrt{R}$) $\sqrt{R\,Pq}$. (Pq. P

$\sqrt{R\,Sq}$. (Sq. S

$\sqrt{PqR+2PRS+SqR}$. $\sqrt{Pq+2PS+Sq}$: P+S Add.

$\sqrt{PqR-2PRS+SqR}$. $\sqrt{Pq-2PS+Sq}$: P−S Sub.

The surds added are $\sqrt{PqR+2PRS+SqR}$:

substracted $\sqrt{PqR-2PRS+SqR}$:

$\sqrt{3})\ \sqrt{27Aq}.\ (9Aq.\quad 3A$

$\qquad \sqrt{3Bq}.\ (1Bq.\quad 1B$

Add: $= \sqrt{27Aq+18BA+Bq}$: $\sqrt{9Aq+6BA+Bq}$: $3A+B$

Sub: $= \sqrt{27Aq-18BA+Bq}$: $\sqrt{9Aq-6BA+Bq}$: $3A-B$

The worke according to the Rule.

$$\begin{array}{r} 3A+B \\ \underline{3A+B} \\ 9Aq+3BA \\ \underline{+3BA+Bq} \\ 9Aq+6BA+Bq \end{array}$$

I conclude $\sqrt{27Aq}+\sqrt{3Bq} = \sqrt{27Aq+18BA+Bq}$ and $\sqrt{27Aq}-\sqrt{3Bq}$ resteth $\sqrt{27Aq-18BA+Bq}$.

2. But if the surds be *Asymmetra*, or have not such a common measure as before, then are they added with + and −.

As $\sqrt{B}$ be to be added to $\sqrt{C}$ it would be $\sqrt{B}+\sqrt{C}$ if subtracted $\sqrt{B}-\sqrt{C}$.

So likewise $\sqrt{B-D}$: $\sqrt{B-C}$ added $\sqrt{B-D}+\sqrt{B-C}$: substracted $\sqrt{B-D}:-\sqrt{B-C}$.

Sect. 2. *Of Multiplication and Division.*

1. If your surds be all of one kinde, then multiply and divide them as is taught before, and prefix before them their former Roote.

As $\sqrt{qA}$: $\sqrt{qB} = \sqrt{qBA}$, & $\sqrt{qA})\ \sqrt{qBA}\ (\sqrt{qB}$.

2. But

2. But if they be not of the same kinde or power, then must they be reduced thus; set the √ as in the Example, and multiply each Quantity as his alterne power or roote doth specifie. Examp. I would multiply √qB by √cC.

B	C = Cq = Bc		Reduced to one denomination are √qcBc
√q	√c = √qc		√qcCq mult. = √qcBcCq

3. If the Characters be compounded, then let the one be reduced to that species, by which it exceedes the other.

Aq	Bq	are made √qq Aqq, and √qq Bq
√q	√qq	

4. If in Multiplication the one be an absolute number, reduce it by Multiplication to the same Roote with the other: As 2×√qB, that is √q4×√qB= √q4B. 2×√cC = √c8×√cC = √c8C.

So √qqD×2 that is √qq D×√qq16 = √qq16D and so of 2.3.4.5. &c.

Hence it is apparent how any Surde may be added to himselfe, that is multipyed by 2. For if I were to add √qB+9D to √qB+9D: it would be √q4B+36D So √q3D in 2 = √q12D. So √c3D added to it selfe = √c24D.

5. If you be to multiply any Roote or Roots the

themselves, if the powers be alike, *viz.* to square the $\surd q5$, then cancell the Roote, and set downe 5 as a plaine Number; so QQ: of $\surd qqB$ is B. But if they be not of the same kinde, divide the Index of the greater by the Index of the lesse, and set downe the power answering the Remaine with your species as Q: the $\surd qqB$ it will be $\surd qB$, because 2) 4 (2. but this onely, if the great Roote be compounded.

CHAP. VIII.

The parts of Numeration in compound surds.

Sect. 1. *Of Addition and Substraction.*

THE Addition and Substraction of these compound surds, is the same with the simple, having respect to the signes.

Examp. *Of Addition.*

Examp. 1. Numbers.	$\surd q27 + \surd q8$
	$\surd q12 + \surd q2$
Summe	$\surd q75 + \surd q18.$

Species

$$\frac{Z}{2} + \surd\,\frac{Zq - 4P}{4}.$$

$$\frac{Z}{2} - \surd\,\frac{Zq - 4P}{4}.$$

Summe $Z.$

$$Z+\sqrt{SR}$$
$$P+\sqrt{ZE}$$
$$Z+P+\sqrt{SR}+\sqrt{ZE}.$$

$$\sqrt{q}Zq-4P: \qquad \sqrt{c}Zq:+5P:$$
$$\sqrt{q}Zq-4P: \qquad \sqrt{c}Zq:-5P:$$
$$\text{sum } \sqrt{q}4Zq\cdot16P: \qquad \text{sum } \sqrt{c}8Zq.$$

Examp. Of Substraction.

1. $\frac{Z}{2}+\sqrt{\frac{Zq-4P:}{4}}$

$\frac{Z}{2}-\sqrt{\frac{Zq-4P:}{4}}$ In Numbers $\sqrt{q}$ 0+16 / $\sqrt{q}$ 320·8 / 24−$\sqrt{q}$

$\sqrt{\frac{4Zq-16P:}{4}}$ $\quad Z+\sqrt{SR}$ / $P+\sqrt{ZE}$

that is $\sqrt{Zq-4P}$. $\quad Z-P+\sqrt{SR}-\sqrt{ZE}.$

2. Addition and, Substraction in universall s
if they be of divers denominations is as in C
pounds, seting before the last the universal signe: I
they be of like denomination, then they are adde
substracted after this manner, adde your two sur
gether, then multiply them, double the Produc
multiply it by $\sqrt{4}$ q set the first Addition and last
duct downe with the signe of Addition if you
but with — if you substract.

Ex. In Numbers and Species.

$\surd 12 + \surd 6$
$\surd 12 - \surd 6$

24 the Add:

$144 - 6 = \surd 138$ the Mul.

and $\surd 138 \times \surd 4 = \surd 552$

therefore $\surd : 24 + \surd 552$ the Add.

$$\left.\begin{array}{l} \surd \frac{Z}{2} + \surd \frac{Zq - 4Pq:}{4} \\ \surd \frac{Z}{2} - \surd \frac{Zq - 4Pq:}{4} \end{array}\right\} \text{To be added.}$$

Addition.

$\frac{Zq}{4} - \frac{Zq - 4Pq}{4} = Pq$ the Mul.

$\surd Pq$ in $\surd 4$ is $\surd 4Pq$.

$\surd : Z + \surd 4Pq = \surd : Z + 2P : =$ the Addition

$\surd : Z - \surd 4Pq = \surd Z - 2P =$ the Substraction.

$$\text{To be ad.}\left\{\begin{array}{l} \surd \frac{2Zq - 4Pq}{4} + \surd \frac{4Zqq - 16PqZq}{16} \\ \frac{2Zq - 4Pq}{4} - \surd \frac{4Zqq - 16PqZq}{16} \end{array}\right.$$

$Zq - 2Pq$ the Addition.

$$\frac{4Z_{q}q - 16Z_{q}Pq + 16Pqq}{16} - \frac{4Z_{q}q - 16PqZ_{q}}{16}$$

That is $\frac{4Z_{q}q - 16Z_{q}Pq + 16Pqq - 4Z_{q}q - 16PqZ_{q}}{16}$

And by cancelling the contrary Species resteth only $Pqq \times 4 = \sqrt{4Pqq} = 2Pq$.

The Addition thereof is $\sqrt{:\, Z_{q} - 2Pq + 2Pq} = \sqrt{:\, Z_{q}} = Z$.

The Substraction is $\sqrt{Z_{q} - 2Pq - 2Pq} = \sqrt{:\, Z_{q} - 4Pq}$:

Another Example.

$$\left.\begin{array}{l} \sqrt{:\, \sqrt{\frac{Xq+4Pq:}{4}}} + \frac{X}{2} \\ \sqrt{:\, \sqrt{\frac{Xq+4Pq}{4}}} - \frac{X}{2} \\ \hline \sqrt{Xq+4Pq} \text{ the Ad.} \end{array}\right\} \begin{array}{l} \frac{Xq+4Pq}{4} - \frac{Xq}{4} = Pq \text{ the Mul.} \\ \text{and } Pq \times 4 = \sqrt{4Pq} = 2P. \end{array}$$

Therefore the Addition is $= \sqrt{:\, \sqrt{Xqq + Pq:} + 2P}$

The Substraction is $= \sqrt{:\, \sqrt{Xqq + Pq:} - 2P}$

Sect. 2. Of Multiplication and Division.

1. There is no great difficulty in this, observing the signes, and in universalls keeping the right signe for the Product.

Ex-

Examples of Multiplication.

$$\frac{Z}{2}+\sqrt{\frac{Zq-4P:}{4}}$$

$$\frac{Z}{2}+\sqrt{\frac{Zq-4P:}{4}}$$

$$\frac{Zq}{4}+\frac{Zq-4P}{4}=\frac{2Zq-4P}{4}\text{ The squares.}$$

$$\left.\begin{array}{l}\sqrt{\dfrac{Zqq-4ZqP}{16}}\times 4=\\[2ex] =\dfrac{4Zqq-16ZqP}{16}\end{array}\right\}\text{The Rectangles.}$$

$$\frac{2Zq-4P}{4}+\sqrt{:\frac{4Zqq-16ZqP}{16}}:\text{ The Product.}$$

$$\sqrt{:}\ \frac{Z}{2} + \sqrt{\frac{Z q - 4Pq:}{4}}$$

$$\sqrt{:}\ \frac{Z}{2} - \sqrt{\frac{Z q - 4Pq:}{4}}$$

$$\sqrt{:}\ \frac{Z q}{4} - \frac{Z q - 4Pq}{4} = \sqrt{:}\ Pq = P.$$

$$\sqrt{:}\ \frac{Z}{2} + \sqrt{\frac{Z p - 4Pq:}{4}}$$

$$\sqrt{:}\ \frac{Z}{2} - \sqrt{\frac{Z q - 4Pq:}{4}}$$

$$\frac{Z q}{4} + \frac{Z q - 4Pq}{4} = \frac{2Z q - 4Pq}{4} \text{ the sq:}$$

$$\frac{4Z qq - 16Z qPq}{16} = 2 \text{ Rect.}$$

the Prod: $\sqrt{:}\ \frac{2Z q - 4Pq}{4} + \sqrt{\frac{4Z qq - Z 16 qPq}{16.}}$

2. Now as in simple surds, if you be to multiply a universall surd by himselfe, you are to blot out the Roote and make it a Compound.

3. If you be to multiply an universall surd by a whole quantity or other roote, you must reduce it to the same denomination, as if you be to multiply $\sqrt{:}\ \frac{Z}{2} + \sqrt{\frac{Z q - 4Pq}{4}}$ by R; you must reduce R to Rq and then to Rqq as followeth.

$$\sqrt{:}\frac{ZRq}{2}+\sqrt{\frac{ZqRqq-4PqRqq}{4}}$$

$\sqrt{q}: 23+\sqrt{44}$ $\sqrt{q}16+\sqrt{q}9$

$\sqrt{q}16+\sqrt{q}9$ $\sqrt{q}16+\sqrt{q}9$

$\sqrt{q}25+\sqrt{576}$ $16+9$

$\sqrt{q}23+\sqrt{4}$ $\sqrt{144} = \sqrt{576}$

$\sqrt{:}25+\sqrt{576}$

$\sqrt{:}575+\sqrt{q}2304+\sqrt{q}2500+\sqrt{q}304704$

that is $575+48+50+552 = 1225$.

$\sqrt{:}1225 = 35$.

Ex. Of Division in Numbers.

$\sqrt{q}3+\sqrt{q}5+\sqrt{q}6)$ 160 $(\sqrt{q}60+2$

$(\sqrt{q}60-2$

that is 56.

$\sqrt{q}3+\sqrt{5}+\sqrt{q}6$

$\sqrt{q}3+\sqrt{q}5-\sqrt{q}6$

$-\sqrt{q}18-\sqrt{q}30-6$

$+\sqrt{q}15+5+\sqrt{q}30$

$+3+\sqrt{q}15+\sqrt{q}18$

Sum $\sqrt{q}60+2$.

$\sqrt{q}5)$ $\sqrt{q}: 13+\sqrt{q}7$ $(2+\sqrt{\ }$

The squares being taken and universall signe cleared.

Sect. 3. Of the Extraction of the Rootes of Binomialls and Residuals Universall.

1. Take the difference of the squares of the parts of the surd to be multiplyed in it selfe, the Roote whereof adde to the greatest Particle, and take it from it: Then the Rootes of halfe those Products being carefully set downe according to the due signe, expresseth the true Rootes.

Ex. To take the Roote of $\sqrt{:}\ 8 + \sqrt{48}$: 64 & 48 are the particular squares, 16 is the difference, 4 the Roote of the *Difference*,

4 being added to the greater 8 = 12 $\frac{1}{2}$ 6. 4 being subtracted = 4 the $\frac{1}{2}$ is 2. } Therefore I say $\sqrt{q}6 + \sqrt{2}$ is the Root of this *Binomial*,

and $\sqrt{q}6 - \sqrt{2}$ the Roote of the *Residuall*.

The square root of $\sqrt{:}\ \frac{Z}{2} + \sqrt{q}\ \frac{Zq - 4Pq}{4}$ is gotten thus.

$\frac{Zq}{4}$ $\frac{Zq-4Pq}{4}$ the sq. Pq the Difference, Roote is P, } $P + \frac{Z}{2} = \frac{Z+2P}{2}$ $\frac{1}{2}$ is $\frac{Z+2P}{4}$; $P - + \frac{Z}{2} = \frac{Z-2P}{2}$ $\frac{1}{2}$ is $\frac{Z-2P}{4}$

The Root of the *Binomiall* is $\sqrt{q}\ \frac{Z+2P}{4} + Zq\ \frac{Z-2P}{4}$.

The Roote of the *Residuall* is $\sqrt{q}\ \frac{Y+2P}{4} - Zq\ \frac{Z-2P}{4}$.

Thus have wee gone through all the simple Elements in species, now come wee to the comparative part, it consisting in the great Rule of *Algebra* or *Equation*, the mystery of the whole Art of Arithmetick, by helpe whereof all obscure and darke Questions are resolved, which without it would never by the ordinary Arithmetick be satisfied ; and therefore for the true understanding heereof, it will be requisite to distinguish it into these parts : First the *definition* of the Rule, *Invention* of the Equation, and *Composition* of the Question, the *Reduction* and severall qualities of it, and its *Resolution* ; and then wee will come to shew the use of it in the easie resolution of any Arithmeticall Question, that is possible to be resolved : Lastly to shew some hidden mysteries in the *Expressions*, *Transmutations*, and *Composures* of those high difficult affected Æquations.

CHAP. IX.

Of Æquation, *and first of the* Invention *or finding out of it.* Sect. 1.

Æquation is a Proportion of *Æquality* betwixt two or more quantities, Numbers or things of the same denomination : As A - C = D supposing A = 12. C = 4. D = 8 for 12 - 4 = 8 ; The Invention

vention and bringing the Question proposed to an *Æquation* is the great difficulty, (for after wee have once certainely found the Æquation, then by Reduction and Resolution the true answer will easily be found) For the easie finding whereof wee are to consider these three Sections following.

Sect. 1. It is very requisite that wee have provided abundance of Analiticall store to be used as occasion may offer, such as Mr. *Oughtred* hath recorded in the XVIII Chap. of his *Clavis*, *Bachetus* before *Diophantus*, *Billy* before his continuall Proportionalls; for how is it possible to resolve a question in proportion unles I know, that in 4 Proportionalls, the Rectangle of the meanes are equall to the Rectangle of the extreames, and such are these that follow?

1. If A. B. C. D. E. be in Arith. Prop: $A - E =$ $\underline{A\ B} + \underline{B - C} + \underline{C\ D}$.

2. If A. M. N L. E $\dot{\dot{::}}$ Q: $\underline{A+E} =$ Q: $\underline{M+L} = \underline{MqLq+}$ $+ 2Nq$. For Q: $M+L = Mq + 2\overline{ML}(=2Nq) + Lq = Mq$ $+ 2Nq + Lq$.

3. If A. M. E $\dot{\dot{::}}$ Q: $\underline{A+E} = Aq + 2Mq + Eq$: for Q $A+E = Aq + 2AE (= 2Mq) + Eq = Aq + 2Mq + Eq$.

4. If A. M. E $\dot{\dot{::}}$ Q: $\underline{A+M+E} = \underline{Aq+Mq+Nq+Eq}$ $+2\square$ ex $\underline{A+M+E}$ in M.

5. If A. M. N. E $\dot{\dot{::}}$ Q: $\underline{A+M+N+E} = \underline{Aq+Mq+Nq+Eq}$ $+2\square$ ex $\underline{A+M+N}$ in $\underline{M+E}$.

6. If A. M. E $::$ $Aq - Eq =$ Q: $\underline{A\ E} + 2q\ \underline{M-A}$

7. $A \times X = Mq - Aq$. 8, $E \times X = Eq - Mq =$ Q: $\underline{A\cdot E}$ $+ \underline{Mq\ Aq}$. 9. $X -$ Q: $\underline{A - E} = 2Mq - 2Aq$. 10. Eq $Mq + Aq =$ Q: $A - E$.

11. $A, M, N, E ∺$ let $Z =$ summe of them all, and let $Z = B+C$ and $B \times C = Q: M+N$. Then $B = A+M$ and $C = N+E$. For $Mq(NA)+2MN(MN+AE)$ $+Nq(ME) = NA+MN+AE+ME$.

12. If $A. M. E.$ $A \times Mq+Eq = M \times Aq+Eq = E \times Aq$ $+Mq$, For $AMq+AEq = MAq+MEq = AqE+MqE$.

13. $AE - Aq = E - A \times A$ and $AE - Eq = A - E \times E$.

14. $Aq+2A+1 = Q: A+1$. 15. $Aq-2A+1 = Q:$ $A-1$. 16. $Ac+2Aq+A+1 =$ Cube $A+1$. 17. If $Aq.$ $AE.$ $Eq ∺$ I say $Aq.$ $AE-E.$ $Q: A-E ∺$ and $Eq.$ $AE-E.$ $Q: A-E ∺$. 18. $Aqq \times Eq = Q: A \times E$.. 19. $Q:$ $Ac+Bc = Q: Ac-Bc+4AcBc$. 19. $Ac-Bc =$ Cube $A-E+\square 3AE \times A-E$. 20. $Ac-Ec =$ Ccc: $A+E-\square 3AE$ $\times X = X$.

Sect. 2. When a question in Arithmetick is asked, you may place for the Quantity or Number sought A (or A joyned with some Number for the ease in working the Question) which according to the Tenor of the Question must be examined, (as though you had it given, and were proving the truth of it) by some of these meanes following, till you can bring it to an *Equation*.

1. Sometimes the sought Numbers or Quantities are many, and that by one Roote put they cannot be knowne, therefore in this case wee put as many vowells or letters for the first magnitude as shall be needfull, and then as many letters you use for the Rootes, so many severall Æquations to finde out the values of these letters in relation to A: and herein great care is

is to be had that after you have found out the values of those second Rootes, you never let them againe enter the Question.

2. Sometimes you must make use of some known Probleme you have in store, which will expedite your worke, and ease you very much.

3. And sometimes you must cavase your Question by Addition, Substraction, Multiplication, and Division, or by some proportion or other Artifice, till you have at length brought out your Æquation, which must be so ordered and reduced, that the species of the Quantity sought with all his parodicall degrees may make up one side, and the given magnitudes the other: the which is taught in the Chapter following.

Sect. 3. It will be no little helpe to the Invention to observe this Rule: That whensoever any Arithmeticall Question, (wherein Numbers are joyned with Materiall Things, as Men, Money, Time, Place, &c.) is propounded; that is a practicall Question: The same be seperated from such things, and resolved in the pure consideration of Numbers: The Examples following will more plainely set out the meaning of this Section.

1. Question. A certaine man went 9 miles a day, another man followes him from the same place, but set forth 10 dayes after, and went 14 miles each day, in how many dayes will he overtake him? The Question *abstractly* put; I demand what Number is that which being multiplyed by B(9) and the Product added to F(

added to C (90) is equall to the same Number multiplyed by D (14)

2. A certaine man hired a servant for a yeare upon this condition, that for every day that his servant laboured he should have a shilling, and for every day he was idle or played, he should lose or discount eight pence: Now at the yeares end the Master was to give the servant nothing, or the servant any thing in the Masters Debt: It is desired to know what dayes the seruant laboured, and what he played?

Abstractly put thus; divide the Number B (365) into two parts, that the one multiplyed by C (12) shall make so much, as the other drawne into D (8.)

3. A Vintner sold 30 Botles of Wine for 210 shillings, whereof some of the Botles were white Wine, others Claret; but he sold one bottle of white Wine for 5s, and a bottle of Claret for 8s. It is demanded what bottles there were of white Wine, and what of Claret?

Abstractly in Numbers thus; divide B (30) into two such parts, that the one part drawne into C (5) the other part into D (8) shall make F (210.)

4. There are two sorts of monies in Number 1000, worth 80 pound, whereof 10 of the one kinde, and 20 of the other are worth 1 pound: It is demanded, what Number there were of both these sorts?

Abstractly in Number thus; divide the Number B (1000) into two such parts, that the one divided by C (10) the other by D (20) the two Quotients shall make F (80.)

5. Suppose *Barwicke* and *London* are distant 228 Miles, out of which two Foot Posts take their journies, and meete the 12 day, but the one went each day one mile further then the other: It is desired to know what miles each went every day?

Abstractly thus; two Numbers are sought whose difference is X (1) That if both of them severally be drawne into B (12.) the summe of the Product will be equall to C (228.)

6. Among two Regiments of Souldiers, whereof the one being Footmen exceeds the other being Horsemen by 300 persons, there was distributed 4000 Crownes, and every Souldier of the Company of Horse, had 3 Crownes more then a Foote Souldier: It is demanded how many Foote Souldiers and how many Horse there were in each Regiment?

Abstractly thus; divide the Number B (4000) by two Numbers, so that the lesser shall be exceeded by the greater by C (300,) And let the Quotient of the first worke be greater then the Quotient of the second by the Number D (3.)

7. One that had bought 100 yards of cloth, another demanded of him what a yard cost him? He answered, for how much lesse I bought 4 yards then for 80 shillings; by so much lesse did I buy 50 yards, then 95 shillings.

Abstractly thus; A Number is sought that being drawne into B (40) and C (50) And from the Products if you substract D (80) and F (95) the Remaines will be equall.

And

And from hence it is apparent conversely how to propound a Question, out of Numbers simply joyned with materiall Substances. And further it is diligently to be noted, that when divers numbers are proposed to bee found, that you seeke out that first, for the which you placed A; as if you did put it for the 1, 2, or 3, then in the resolution of the Æquation it may still signifie the first, second, or third, according to the Position.

And note, that of all the parts of Æquation, this only of the *Invention* is most difficult; for besides the necessity of the knowledge of the Principles of Geometry and Arithmeticke, together with a good judgement, it is very requisite, that hee that desires to bee a learned *Analist*, shall practice the finding out of many hard questions, which hee may both finde from others, and propose to himselfe as occasion may serve.

Section 2. *Of the Reduction of the Æquation, the works Consomitant with it.*

1. To reduce an Æquation, is so to order and dispose it (keeping it still to some equallity) that it may be fit for Resolution, and this is done 5 manner of wayes. 1. By Reduction of the Fractions in an Æquation (if there be any) into whole species. 2. By Transposition. 3. By common depression. 4. By a common division of both sides of the Æquation. 5. By Reduction of either side of the Æquation to the same power, and cancelling the Cossick signes.

Isomeria Viet.

1. If every or any magnitude in either side of the Æquation be a Fraction, or Fractions, wee multiply all the other species in each other Denominator but his owne; and this is nothing but the worke of Fractions, and therefore as occasion serves the greatest common measure may likewise be taken. Examples.

(1) $A+B = \frac{D}{C}$ and $AC+BC = D$.

3. $A + \frac{B}{DC} = B + \frac{5A}{G}$ and $DCGA+BG = BGDC+5DCA$.

4. $\frac{AcBq}{C} + BqGA = \frac{2DqBA}{3} + \frac{CBqAq}{D}$

and $3AcBqD+3BqGDCA = 2DcCBA.$
$+3CBqAqC.$

Antithesis Viet.

2. That the Magnitude or Magnitudes sought may make up the one side of the Æquation, by transposition, you must convert the given Magnitudes, and they sought from the one side to the other, changing the signes.

1. $AC+BC = D$. $AC = D - BC$. 2. $AEq+DCE = GCE$ and $AEq = GCE - DCE$.

3. $DCGA+BG = BGDC+5DCA$ and $DCGA - 5DCA = BGDC - BG$

3. $Aq+BC = DB+AF$ and $Aq+BC-AF = DB$ and $Aq-AF = DB-BC$.

4. $Z_? = 2Aq-2XA+Xq$ and $2Aq-2XA = Z_?-Xq$.

Hypobibasmus Viet.

3. If the Magnitude sought be found in all the Magnitudes on either side of the *Æquation*; then you must by depression or dividing the Magnitudes sought by some Parodicall degree, cleare some species of the Æquation, that the one side may be knowne.

$AC+BAq = DA$ depressed becomes $Aq+BA = D$.

$AqqB+5DCAq = BCAq+DAq$ and $AqB+5DC = BC+D$ and $AqB = BC+D-5DC$.

$AqcD-5BAq = DcAqq$ and $Ac-5B = DcAq$ and $Ac-DcAq = 5B$.

Parabolismus Viet.

4. If the Magnitude sought or his highest power be joyned, with any other species, then must the rest of the species, be divided by that species, and the highest power cleared thereof.

1. $AqB = D+C$ and $Aq = \frac{D+C}{B}$.

2. $AqB-DA = R$ and $Aq-\frac{D}{B}A = \frac{R}{B}$.

5. If any species be expressed in the Æquation by a Cossicke roote, then must the rest be exalted to that power.

(1) $B+C = \sqrt{A}$ and $Bq+2BC+Cq = A$. (2) $B = \sqrt{AB} - D$. and $B+D = \sqrt{AB}$. Then $Bq+2BD+Dq = AB$ and $A = B+2D+\frac{Dq}{B}$. 3. $B+D = \sqrt{:B+\sqrt{D}}$: Then $Bq+2BD+Dq = B+\sqrt{D}$ and $Bq+2BD+Dq-B = \sqrt{D}$ to be reduced as before.

Examples where all the Rules of Reduction are used.

If this Æquation $Zq = \frac{PRq+PSq+2PRS}{RS}$ were given, and suppose the sought Magnitude were S. Then $ZqRS = PRq+PSq+2PRS$, and $ZqRS-2PRS-PSq = PRq$ and $\frac{ZqR-2PR}{P}$ $*S - Sq = Rq$.

Suppose R were sought in the the same Æquation, Then $ZqRS-2PRS-PRq = PSq$ and $\frac{ZqS-2PS}{P}$ $*R - Rq = Sq$.

Suppose P were sought, Then $RqP+SqP+2RSP = ZqRS$ and $P = \frac{ZqRS}{Rq+Sq+2RS}$.

If the given magnitude be equall to 2 surds as $Z = \sqrt{AP}+\sqrt{AS}$ Then $Zq = AP+AS+\sqrt{4AqPS}$. Then $Zq-AP-AS = \sqrt{4AqPS}$ to be resolved as before.

2. These 5 severall parts of Reduction doth not change

change the equality, the 1 doth not change the Æquality, for let $\frac{2}{4} = \frac{3}{6}$ Then 2. 4 : : 3. 6. and because the Product of the meanes are equall to the extreames therefore $4 \times 3 = 2 \times 6$. in species $\frac{A}{B} = \frac{C}{D}$

Then A. B : : C. D and BC = AD.

The 2 doth not alter the Æquality, the same Magnitude being added to both sides.

The 3 and 4 doth not alter the Æquality, both sides of the Æquation being divided by one Magnitude.

The 5 doth not alter the Æquality both sides being squared or multiplyed into themselves.

3. But if the Æquation fall out contrary to the definition, then is it either *Nugatory or impossible*: 1) As if there be an Identicall Aquation thus 3Aq = 3Aq Then may it be resolved by any Number whatsoever, and therefore is it a vaine Question : 2. If the Æquation fall out to be 5Aq = 4Aq. Then is the Question propounded utterly impossible, and cannot be resolved: Likewise if the Question be Aq = 12A - 40.

Because 40 cannot be taken from 36 the square of $6\frac{1}{2}$, therefore likewise is this question impossible to be answered : And the finding out of these fallacies in Æquations is a great Priviledge of this Art: and thus much for the worke accompanying the Æquation.

Sect 3. *Of the Resolution of all sorts of Æquations in Numbers.*

1. After your Æquation is invented and reduced, then the sought Magnitude is either pure, As A = B+C, or secondly it is some *Potestas*, as Aq. Ac. Aqq &c. or thirdly, it is an adfected Æquation, of which last sort there are also two kinds: 1. The Indices of the Æquation are in Arithmeticall Proportion, As Aq+BA = C, that is 0. 1. 2. Aqq - DAq = A, that is 4. 2. 0. or 6. 3. 0. or 8. 4. 0. &c. Or secondly, they are not ascending or descending in such Cossick signes as will make up any part of Arithmeticall Proportionals.

1. If the sought Magnitude be a pure Roote, as A. Then according as the other side of the Æquation doth intimate, *viz.* by *Addition*, *Substraction*, *Multiplication*, or *Division*, you may finde the value thereof: As for Example. $A = \frac{X+Xq}{2X}$ supposing X = 5 and X = 1 as in the Chapter following $\frac{5+1}{2} = \frac{6}{2} =$ 3 = A. in words you must adde the Number intimated by X to the square of the Number intimated by X, and dividing that summe by X doubled, the Quotient is equall to A.

2. If the sought Magnitude be some *Potestas*. Then after you have added, substracted, multiplyed

or divided the other side of the Æquation as the species informe you, then must you exstract the Roote of that Result according to the Cossicke signe annexed to the sought species; As for Aq take the square Root, Ac the Cube Roote, &c.

The extracting of these Rootes, as also the Rootes of all adfected Æquations in Numbers, are most learnedly taught by the oft before mentioned Mr. *Oughtred*, in his *Clavis*, and though by some handled, yet never before him (to the perpetuall credit of this Nation) so plainely, methodically, and succinctly explained, not onely in the true Nature of the Rootes and their Adfections, but in the easie invention of the first and second figures of the Roote, which before was a businesse of very great difficulty, as those who make experiments therein may very readily perceive.

For the more easie and speedy extracting of all sort of Rootes I shall commend to the Reader, and especially to Gentlemen that are willing to studdy this Art, (who as they ought to be encouraged, to proceede with as much ease as can be, so commonly being deterred, by the tediousnesse of practise and difficulty, doe to the great prejudice of it and Artists, not onely leave it off, but deter others from studdying it) these two wayes, *viz.* the practice of Napieres bones, and the use of the Table at the end hereof joyned both together, by helpe whereof I dare affirme to extract a symple or adfected Æquation, and that without the ascribing any thing to memory in halfe the time, if

not

not lesse then any who shall doe it without.

3. The Table annexed to the end of this Booke, containes the squares and cubes of all Numbers under 1000. The squared squares of all Numbers under 500, and the squared Cubes, and Cubed Cubes under 250: There is also all the Multiplees of single squares, Cubes, square squares, and square Cubes annexed, which are of great use likewise in Extraction.

4. If the square Roote of any Number under 1000000. the Cube Roote of any Number under 1000000000, The sqq: Roote of any Number under 62500000000, The sqc Roote of any Number under 9765625000, Or Cube, Cube, Roote under 244140625000000 be demanded, under each respective power seeke out for your Number given, and under A if you hit just of that Number you have the true Roote; as if it be demanded what is the √q of 243049, I finde it be 493. the √qq of 1121513121, I finde it to be 183, &c.

But if your Number fall out betwixt other two Numbers that are truely Radicall, then you may be sure the higher Roote is the Roote of Integers, and then you may continue to get a *Decimall Fractionall* Roote, by joyning 3 Cyphers to the difference; but by what ease and speedy practice, both this and other the many mysteries of these Numbers may be wrought, neither the stinted Bigues of this booke, or other my imployments, will at this time give me leave to set downe.

The

The Extraction of the square Roote by the Bones.

First point every other Figure from your right hand beginning with the last; and finding out of the square bone, the next lesse or equall square to the Figures of the first point to your left hand, set the Index in the Quotient, and substracting the true square from the Number above, to the Remaine adde the next two Figures, and for a Divisor being (2A) double the Quotient, and set the doubled Quotient on the bones, and place them before the square bone, which done, worke as in Division, by seeking a Quotient, and substracting the Number on the bones (which are equivolent to (2AE+Eq) the Gnomon) from the Numbers above; onely observing this difference, that you must still draw downe two Figures, and that every time you must double your Quotient for a New Divisor.

$$\begin{array}{ll} & Aq: \\ & \left.\begin{array}{l} \cdot\cdot\cdot\cdot\\ 2A{:}\,E \\ {:}Eq \end{array}\right\}\text{Gnom.} \\ \text{Divisor} & \end{array}$$

. . . .
10601536 (3256
A) 09Aq
2A6) 160 R:
124 (2AE+Eq.
2A64) 36.5 R:
3225 (2AE +Eq
2A690) 39036 R.
39036 (2AE+Eq
00000

34'00000000 (5'8308
A) 25 Aq
2A10) 900 R:
864 (2AE+Eq
2A116) 3600 R:
3489 (2AE +Eq
2A1166) 11100 R:
2A11660) 1110000 R:
9328642AE +Eq
177136

To Extract the Cube Roote.

You are to observe the same worke as in the square Root with these Differences.

Ac
....

Divisor $\left\{ \begin{array}{l} 3Aq{:}\,E \\ 3\ \ A{:}\,Eq \\ \quad\ {:}\,Ec \end{array} \right\}$ Gnom.

1. That you point every third Figure. 2. That you trible the square of the Quotient (which is nothing but to finde the square of the Quotient in the Table of squares, and multiply it by 3) for every new Divisor, which must be set before the Cube bone; and that after you have found out a Quotient, you must to the Numbers on the bones, adde the trible of the Quotient multiplyed by the square of the last Figure found (which by another set of bones had in readinesse, may easily be done placing it one place towards the left hand, that is (3AEq) A signifying all the Quotient, and E the last figure, and if then the Gnomon (that is 3AqE+3AEq+Ec) doe exceed the Figures, you must abate one from the last Quotient: The which by observing, the worke may easily be avoyded.

The Analysis of the Cube.

```
        ·  ·  ·
        146363183 (527.
        125.    Ac
3Aq...75)  21363  Resid.
        15008..3AqE+Ec
           60.. 3AEq
        15068
3Aq..8112)  5755183  Resid.
        5678743..3AqE+Ec
            7644 .. 3AEq
        5755183
              00  Resid.
```

To extract the squared square Roote.

The extraction onely alters in the points, which must be every fourth Figure, the Divisor which must now stand of two sets of bones, and upon the one the Cube of the Quotient drawne into 4, and the square of the Quotient drawne into 6, and lastly, that after you have estimated a new Quotient, you must adde to the Number on the bones these two summes viz. (6AqEq) the square of the Quotient in 6, in the square of the last Figure set 2 places, short from the summe of the bones, and the Quotient in 4 in the Cube

Ex-

And

The Analysis of the Byquadratic Power.

	1071	1675	9488	0904	1761	(57209	
	625	Aqq					
	446	1675					
500 (1))	......	......					Aqq.
150 (2))	350	2401	4AcE+	Eqq			......
	73	50	6AqEq				4Ac: E
	6	860	4AEc				6Aq:Eq
	~~430~~	~~6001~~	Gno:				4A :Ec
	15	5674	9488				.Eqq
740772 (1))	14	8154	4015	4AcE+	Eqq		
19494 (2))		779	76	6AqEq			
		1	824	4AEc			
	~~14~~	~~8935~~	~~0856~~	Gno:			
74879332528 (1))		6738	9632	0904			
1963104 (2))		6738	9632	0904	1761		
		6737	3729	2800	1561	4AcE+Eqq.	
74879332528000)		1	5901	1424	00	6AqEq	
			1	6679	520	4AEc	
		~~6738~~	~~9632~~	~~0904~~	~~1761~~		

Place this Table in the second Alphabet Fol. 63, at this Marke, ¶

be of the last Figure set one place short : An Example will make the matter more plaine.

Aqq

....

$$\text{Divisor}\left\{\begin{array}{l} 4Ac: E \\ 6Aq: Eq \\ 4A: Ec \end{array}\right\}\text{Gnomon.}$$

:Eqq

And note that all along the operation, I make u of the Table annexed at the end hereof, as when I ha 572 to be cubed and multiplyed by 4, I find the Cub of 572, which is 186149248, and multiply it b 4 == 744596992, which I doe with the squares, & which is no small ease in the worke.

The Rootes of the other powers are after the sam manner extracted, observing their points and interme diate species, which by Chapter 4, Sect. 5. are easil made up annexing the *Uncia* to the A, and note th all the *Uncia* and A are still the Divisor, the *Un* A and E the *Gnomon*: and that further the Numbe belonging the highest powers will fall directly und the points, and the intermediate powers will fall far short of either, as they are in distance from the

Aqc:
....

Divisor { Aqq: E
Ac : Eq
Aq : Ec
A : Eqq
Eqc } The Gnomon.

5. If the one side of the Æquation be an adfect Æquation, and yet the Indexes thereof are in Arit meticall Proportion, then it being some of these th follow. ZA - Aq == Æ. Aq - XA == Æ. or Eq†X == Æ.

Th

The Resolutions by Chap. XVI. Sect. 9. are

$$1.\ \frac{Z}{2} \pm \sqrt{q}: \frac{Zq - 4Æ}{4} := \begin{matrix} A \\ E \end{matrix}$$

$$2\sqrt{q}: \frac{Xq + 4Æ}{4} : \pm \frac{X}{2} = \begin{matrix} A \\ E \end{matrix}$$

But if it happen the midle species to be a square or a Cube, then must it be the universall square Roote or Cube Roote: As if it be $Z, Aq - Aqq = Æq$ the resolution is $\sqrt{q}: \frac{Z,}{2} \pm \sqrt{q}\ \frac{Z,q - 4Pq}{4} := \begin{matrix} A \\ E \end{matrix}$

The resolution into Numbers is as easie, for if the highest species be negative, then the Rule is, take halfe of the Coefficient and adde to it, or substract from it the square Roote of the square of the Coefficient, lessened by the other side of the Æquation in 4 and divided by 4, the summe or difference is the greater or lesser Roote, and so of the other Rules, onely if the middle species be a square Number, then you must extract the square Roote, &c.

6. If the one side of the Æquation be adfected, and the Indices not in Geometricall Proportion, you must extract the Roote thereof after the Rules delivered in the *Clavis Math.* which were a great injury to the booke here to insert; before I saw that piece I had done somewhat for the easie extraction therof; the

which I now omit to set down: I shall only give you a Breviat how to make the Canon for extracting the Roote: If this Æquation Aqc - DAq + CA = B were proposed.

Aqc	- DAq	CA
• • • • •	• • • • •	• • • •
Aqq : E	- D2A : E	C : E
Ac : Eq	- D ·· : Eq	
Aq : Ec		
A : Eqq		
Eqc		

The first Divisor is C - D.

The first Ab. Aqc + CA - DAq.

The second Divisor is 5 Aqq + 10 Ac + 10 Aq + 5 A + C - D2A - D.

The second Ab. 5 AqqE + 10 AcEq + 10 AqEc + 5 AEqq + Eqc + CE - D2AE - DEq.

The pricks and intermediate species follow the difference, as was before set downe in the pure Roots.

And if you observe in the Extraction the Powers, you may out of the Table at the end hereof, take out the Numbers answering with great ease: And thus have we gone through the parts of Æquation: now come wee to practise them and the former Rules by Examples.

Chap.

CHAP. X.

Containing severall considerations of two Numbers and Questions thereof (deduced from the Chap. XI. *of the* Clavis Math. *) wherein all the former Rules in this Booke are practised, being usefull for the managing of an Æquation.*

THE generall varieties go upon these Questions, that having any two of the Numbers mentioned Chap. 1. Sect. 16. thereof, *viz.* A. E. Z. X. P. $\frac{R.}{S}$ Z. X. Z is X to finde out any of the rest.

Question 1. Having two Numbers given, whereof A the greater and E the lesser to finde their summe, Difference, Rectangle, Proportion, summe of squares and differences of squares, *viz.* Z. X. P $\frac{R.}{S}$ Z. X.

A	is	A given
E		E given
Z		A+E by Addition
X		A - E by Substraction
P		AE by Multip:
$\frac{R}{S}$		$\frac{RE}{A}$ By the Golden Rule: A.E : : R.
Aq		Aq. by squaring A and E
Eq		Eq
Z		Aq+Eq. By Add:
X		Aq - Eq By Sub:

$Z = Ac+Ec. \quad X = Ac - Ec.$

Quest. 2. Having the greater of two Numbers given, and the summe of them, *viz.* A and Z to finde out E. X E. S. Aq. Eq Z. X: First I seeke E by the last, for A+E = Z then E = Z - A: the rest as followeth.

A	is	A
E		Z - A.
X		2A - Z
P		ZA - Aq
S		$\frac{ZR - RA}{A}$
Aq		Aq
Eq		Zq - 2ZA+Aq
Z		Zq - 2ZA+2Aq
X		2ZA - Zq.

And in this worke, for examining the truth the species may be put into Numbers, make $A = 3$ and $E = 2$. As $X = 2ZA - Zq$ in Numbers $5 = 30 - 25$.

Question 3. Having Z and E, to finde A, X, P, S, Z, X.

First I finde A thus, by the first $Z = A+E$, $A = Z - E$. therefore

A		Z - E
E		E
X		Z - 2E
P		ZE - Eq.
S	is	RE / Z - E
Aq		Zq - 2ZE+Eq
Eq		Eq
Z		Zq - 2ZE+2Eq
X		Zq - 2ZE.

Quest. 4. Having X and A given, to finde E, Z P, S, Z, X.

First I finde E by the 1. for if X = A - E, E = A - X and

A		A
E		A - X
Z		2A - X
P	is	Aq - XA
S		$\frac{RA - XR}{A}$
Aq		Aq
Eq		Aq - 2XA + Xq
Z		2Aq - 2XA + Xq
X		2XA - Xq.

Question 5. Having X and E given, to finde A, Z, P, S, Z, X.

First *per* 1. X = A - E. A = X+E. therefore

A		X+E
E		E
Z		X+2E
P		XE+Eq
S	is	$\frac{RE}{X+E}$
Aq		Xq+2XE+Eq
Eq		Eq
Z		Xq+2XE+2Eq
X		Xq+2XE.

Question

Question 6. Having P and A given, to finde out E, Z, X, S, Z and X.

First (*per* 1) if $P = AE.$ $E = \frac{P}{A}.$

A		A
E		$\frac{P}{A}$
Z		$\frac{Aq + P}{A}$
X		$\frac{Aq - P}{A}$
S	is	$\frac{RP}{Aq}$
Aq		Aq
Eq		$\frac{Pq}{Aq}$
Z		$\frac{Aqq + Pq}{Aq}$
X		$\frac{Aqq - Pq}{Aq}$

Question 7. Having P and E given to finde out A, Z, X, S, Z, X.

First if $P = AE$ then $A = \frac{P}{E}$ therefore

A		$\frac{P}{E}$
E		E
Z		$\frac{Eq+P}{E}$
X		$\frac{Eq-P}{E}$
S	is	$\frac{REq}{P}$
Aq		$\frac{Pq}{Eq}$
Eq		Eq
Z		$\frac{Pq+Eqq}{Eq}$
X		$\frac{Pq-Eqq}{Eqq}$

Question 8. Having $\frac{R}{S}$ and A given to finde out E, Z, X, P, Z, and X.

First I finde E thus $\frac{R}{S} = \frac{A}{E}$ and $SA = RE$ therefore $E = \frac{SA}{R}$.

A		A
E		$\frac{SA}{R}$
Z		$\frac{RA+SA}{R}$
X		$\frac{RA-SA}{R}$
P	is	$\frac{SAq}{R}$
Aq		Aq
Eq		$\frac{SqAq}{Rq}$
Z		$\frac{RqAq+SqAq}{Rq}$
X		$\frac{RqAq-SqAq}{Rq}$

Question 9. Having $\frac{R}{s}$ and E given to finde out A, Z, X, P, Z, and X.

First $\frac{R}{s} = \frac{A}{E}$ therefore $A = \frac{RE}{s}$.

A	$\frac{RE}{S}$
E	E
Z	$\frac{RE+SE}{S}$
X	$\frac{RE-SE}{S}$
P	$\frac{REq}{S}$
Aq	$\frac{RqEq}{Sq}$
Eq	Eq
Z,	$\frac{RqEq+SqEq}{Sq}$
X	$\frac{RqEq-SqEq}{Sq}$

Question 10.

Question 10. Having the Z, and A given to finde out E, Z, X, P, S, X.

First I find E thus Z, = Aq+Eq E = √Z, - Aq:

A		A
E		√Z, - Aq:
Z		A+√Z - Aq:
X		A - √Z, - Aq
P	is	√Z,Aq - Aqq.
S		$\frac{\sqrt{Z,Rq - RqAq:}}{Aq}$
Aq		Aq
Eq		Z, - Aq
X		2Aq - Z,.

Question 11. Having Z, and E given to finde out A, Z, X, P, S. X.

First if Z, = Aq+Eq therefore A = √Z, - Eq.

A		√Z, - Eq
E		E
Z		√Z, - Eq: +E
X		√Z, - Eq: - E
P	is	√Z,Eq - Eqq:
S		$\sqrt{\frac{RqEq}{Z, - Eq}}$
Aq		Z, - Eq
Eq		Eq
X		Z, - 2Eq.

Question 12.

Question 12. Having X, and A given to finde E, Z, X, P, S, Z, first finde E thus, if X = Aq Eq. and E = √Aq - X,: therefore

A		A
E		√Aq - X,:
Z		A + √Aq - X:
X		A - √Aq - X,:
P		√Aqq - X,Aq:
S	is	$\sqrt{\frac{AqRq - X,Rq}{Aq}}$
Aq		Aq
Eq		Aq - X,
Z,		2Aq - X,

Question 13. Having X, and E given to finde out A, Z, X, P, S, Z,

First I finde out A thus, X, = AqEq

A = √X,+Eq Therefore.

A		√X,+Eq:
E		E
Z		√X,+Eq: + E:
X	is	√X,+Eq: - E.
P		√X,Eq+Eqq:
S		$\sqrt{\frac{RqEq}{X,+Eq}}$
Aq		X,+Eq
Eq		Eq
Z,		X,+2Eq.

Question 14.

Question 14. Having 'Z and A given to finde out E, Z, X P, S, Z, and X,:

First if 'Z = Ac+Ec then E = √c'Z - Ac.

A	A.
E	√c:'Z - Ac:
Z	A+√c'Z - Ac:
X	A - √c'Z - Ac:
P	√c'ZAc - Acc:
S	$\sqrt{\dfrac{\text{'ZRc - RcAc}}{\text{Ac}}}$
Aq	Aq
Eq	√c'Zq - 2'ZAc+Acc:
Z,	Aq+√c'Zq - 2'ZA+Acc:
X,	Aq - √c'Zq - 2ZA+Acc.

The same worke will follow, supposing 'Z and E given to finde the rest.

Question 15.

Question 15. Having the 'X and E given to finde E, Z, X, P, S, 'Z and 'X.

First I finde A thus 'X = Ac - Ec. Then A = √c'X+Ec.

A	√c'X+Ec:
E	E
Z	√c'X+Ec: +E
X	√c'X+Ec: - E
P	√'XEc+Ecc:
S	√ RcEc / 'X+Ec
Aq	√c'Xq+2'XEc+Ecc
Eq	Eq.
Z	√c'Xq+2'XEc+Ecc: +Eq
X	√c'Xq+2'XEcEcc: - +Eq.

The same worke if 'X and E were given.

Question 16

Question 16. Having the summe and difference of wo Numbers given to finde out the Numbers, that is, aving Z and X given to finde out A, E, P, S, Z, and X.

First I finde A and E by the second and third queions, supposing them given, for if $X = 2A - Z$ herefore $X + Z = 2A$ and $A = \frac{Z+X}{2}$ and by the third Question $E = \frac{Z-X}{2}$ therefore

A	$\frac{Z+X}{2}$
E	$\frac{Z-X}{2}$
P	$\frac{Zq-Xq}{4}$
S	$\frac{ZR-XR}{Z+X}$
Aq	$\frac{Zq+2ZX+Xq}{4}$
Eq	$\frac{Zq-2ZX+Xq}{4}$
Z,	$\frac{Zq+Xq}{2}$
X,	ZX.

Question 17. Having the summe of two Number and the Plaine or Rectangle of them, *viz.* Z and P t finde out A, E, X, S, Z, and X,.

First I seeke out A and E by supposing them give by the sixt Question.

For $Z = \frac{Aq+P}{A}$ therefore $ZA = Aq+P$. and ZA

$Aq = P$ and $\frac{Z}{2} + \sqrt{\frac{Zq-4P}{4}} = \frac{A}{E.}$

therefore

A		$\frac{Z}{2} + \sqrt{\frac{Zq-4P:}{4}}$
E		$\frac{Z}{2} - \sqrt{\frac{Zq-4P:}{4}}$
X		$\sqrt{Zq-4P:}$
S	is	$\frac{2ZR - \sqrt{q\,ZqRq - 4PRq}}{2Z + \sqrt{:4Zq-16P.}}$
Aq		$\frac{2Zq-4P}{4} + \sqrt{\frac{4Zqq-16ZqP}{16}}$
Eq		$\frac{2Zq-4P}{4} - \sqrt{\frac{4Zqq-16ZqP}{16}}$
Z,		$Zq - 2P$
X,		$\sqrt{Zqq-4ZqP:}$

Question 1

Question 18. Having the summe of two Numbers, and the Proportion of them, that is, having Z and $\frac{R}{S}$ to find A, E, X, P, Z, and X.

First by the 8 and 9 I finde out A and E by supposing them given thus $Z = \frac{RA + SA}{R}$ and $ZR = RA + SA$ and $A = \frac{ZR}{R+S}$ and $E = \frac{ZS}{R+S}$

Wherefore

A	is	$\frac{ZR}{R+S}$
E		$\frac{ZS}{R+S}$
X		$\frac{ZR - ZS}{R+S}$
P		$\frac{ZqRS}{Rq+2RS+Sq}$
Aq		$\frac{ZqRq}{Rq+2RS+Sq}$
Eq		$\frac{ZqSq}{Rq+2RS+Sq}$
Z		$\frac{ZqRq+ZqSq}{Rq+2RS+Sq}$
X		$\frac{ZqRq - ZqSq}{Rq+2RS+Sq}$.

 Quest.

Question 19. Having the summe of two Numbers, and the summe of their squares, that is Z and Z, given to finde A, E, X, P, S X.

First by the 10 and 11 I finde by supposition A and E thus $Z = A + \sqrt{Z, - Aq}$. and $Z - A = \sqrt{Z - Aq}$ and $Zq - 2ZA + Aq = Z, - Aq$ and $2ZA - Aq = Zq - Z,$.

and $\frac{Z}{2} \pm \sqrt{\frac{Zq - 2Zq + 2Z,}{4}} = \frac{A}{E.}$

Therefore

$$A \text{ is } \frac{Z}{2} + \sqrt{\frac{Zq - 2Z + 2Z,}{4}}$$

$$E \text{ is } \frac{Z}{2} - \sqrt{\frac{Zq - 2Z + 2Z,}{4}}$$

$$X \text{ is } \sqrt{2Z, - Zq.}$$

$$P \text{ is } \frac{Zq}{4} - \sqrt{:\frac{2Z - Zq.}{4}}$$

$$S \text{ is } \frac{2ZR - \sqrt{4ZqRq - 8ZqRq + 8ZRq:}}{2Z + \sqrt{4Zq - 8Zq + 8Z,:}}$$

$$Aq \text{ is } \frac{Z,}{2} + \sqrt{\frac{2Z,Zq - Zqq}{4.}}$$

$$Eq \text{ is } \frac{Z}{2} - \sqrt{\frac{2Z,Zq - Zqq}{4}}$$

$$X \text{ is } \sqrt{2Z\,Zq - Zqq.}$$

Question 20.

Question 20. Having the summe of two Numbers and the difference of the squares given, that is Z and Z, to finde A, E, X, P, S. X,.

First I finde A and E by the 1 thus. $X, = 2ZA - Zq$ and $X, + Zq = 2ZA$ and $A = \frac{Zq + X,}{2Z}$ and $E = \frac{Zq - X,}{2Z,}$

Therefore

A	$\frac{Zq + X,}{2Z}$
E	$\frac{Zq - X,}{2Z}$
X	$\frac{X,}{Z}$
P	$\frac{Zqq - X,q}{4Zq}$
S	$\frac{ZqR - X,R}{Zq + X,}$
Aq	$\frac{Zqq + 2ZqX, + X,q}{4Zq}$
Eq	$\frac{Zqq - 2ZqX, + X,q}{4Zq}$
Z,;	$\frac{2Zqq + 2X,q}{4Zq}$

Question 21. Having the summe of two Numbers, and the summe of their Cubes to finde the sayd two Numbers, that is having Z and Z, given to finde A, E.

First I finde by the 14, that by a supposition, $Z = A + \sqrt{c}\, Z, - Ac$ and $Z - A = \sqrt{c}\, Z, - Ac$. Therefore by Reduction $Zc - 3ZqA + 3ZAq - Ac = {}^{\backprime}Z - Ac$. and $3ZqA - 3ZAq = Zc - {}^{\backprime}Z$, Therefore $ZA - Aq = \frac{Zc - {}^{\backprime}Z.}{3Z.}$

Wherefore

$$\frac{Z}{2} \pm \sqrt{q}\, \frac{Zq}{4} - \frac{Zc - {}^{\backprime}Z}{12Z} = \frac{A}{E.}$$

Question 22.

Question 22. Having the difference of two Numbers, and the Rectangle given, *viz.* X and P, to finde out A, E, Z, S. Z, X,.

First I finde out A and E by the 4. P $=$ Aq - XA

and $\sqrt{q}\ \dfrac{Xq+4P}{4} : \pm \dfrac{X}{2} = \dfrac{A}{E.}$

Therefore

A	$\dfrac{Xq+4P:}{4} + \dfrac{X}{2}$
E	$\dfrac{Xq+4P}{4} : - \dfrac{X}{2}$
Z	$\sqrt{}Xq+4P:$
S	$\dfrac{\sqrt{}4XqRq+16PRq: - 2XR}{\sqrt{}4Xq+16P:+2X.}$
Aq	$\dfrac{\sqrt{}Xqq+4XP}{4} : + \dfrac{Xq+2P}{2}$
Eq	$\dfrac{\sqrt{}Xqq+4XP}{4} : - \dfrac{Xq+2P}{2}$
Z,	Xq+2P.
X.	$\sqrt{}Xqq+4XP:$

Question 23. Having X and $\frac{R}{s}$ given to finde out A, E, Z, P, Z, and X.

First by the 8 *Q:* I finde out A and E thus by supposition.

$$S = \frac{RA - XR}{A} \text{ and } RA - SA = XR \text{ and } A = \frac{XR}{R-S}$$

and $E = \frac{SX}{R-S}$: Therefore

A		$\frac{RX}{R-S}$
E		$\frac{SX}{R-S}$
Z		$\frac{RX+SX}{R-S}$
P	is	$\frac{RSXq}{Rq-2RS+Sq}$
Aq		$\frac{RqXq}{Rq-2RS+Sq}$
Eq		$\frac{SqXq}{Rq-2RS+Sq}$
Z.		$\frac{RqXq+SqXq}{Rq-2RS+Sq}$
X.		$\frac{RqXq-SqXq}{Rq-2RS+Sq}$

Question 24. Having X and Z, to find A,E,Z,P,S, and X.

First I finde out A and E by the 4 *Question* $Z, = 2Aq - 2XA + Xq$ and $Aq - XA = \frac{Z, - X}{2}$

wherefore $\sqrt{\frac{2Z, - Xq}{4}} : \pm \frac{X}{2} = \frac{A}{E}$ and

A	$\sqrt{\frac{2Z, - Xq}{4}} : + \frac{X}{2}$
E	$\sqrt{\frac{2Z, - Xq}{4}} : - \frac{X}{2}$
Z	$\sqrt{2Z, - Xq}$
P	$\frac{Z, - Xq}{2}$
S	$\frac{\sqrt{4Z,Rq - 2XqRq} : - XR}{\sqrt{4Z, - 2Xq} : + X}$
Aq	$\frac{Z,}{2} + \sqrt{\frac{2X,Xq - Xqq:}{4}}$
Eq	$\frac{Z,}{2} - \sqrt{\frac{2X,Xq - Xqq}{4}}$
X,	$\sqrt{2Z,Xq - qq}$

Question 25. Having the difference of two Numbers given, and the difference of the squares, *viz.* X, and X, finde out A, E, Z, P. $\frac{R}{S}$ Z,.

First by supposition I finde out A and E by the fourth, $X, = 2XA - Xq$, and $A = \frac{X, + Xq}{2X}$ $E = \frac{X, - Xq}{2X}$

Therefore.

A	is	$\frac{X, + Xq}{2X}$
E		$\frac{X, - X}{2X}$
Z		$\frac{2X}{2X,}$
P		$\frac{X,q - Xqq}{4Xq}$
S		$\frac{XR - XqR}{X, + Xq}$
Aq		$\frac{Xq + 2X,Xq + Xqq}{4Xq}$
Eq		$Xq - 2X,Xq + Xqq$
Z		$\frac{Xq + Xqq.}{2Xq}$

Question 26

Question 26. Having the summe of the Cubes, and the difference of the Numbers given, *viz.* X and 'Z to finde out the Numbers themselves.

First by the 14. X = A - √c'Z - Ac: and A - X = √c'Z - Ac. and Ac - 3AqX+3AXq - Xc = 'Z - Ac, Therefore 2Ac - 3XAq+3XqA = 'Z+Xc and Ac - $\frac{3X}{2}$ *Aq+ $\frac{3Xq}{2}$ *A = $\frac{'Z+Xc}{2}$ which is an adfected Æquation; now suppose $\frac{3X}{2}$ = B and $\frac{3Xq}{2}$ = D

Then the Æquation is Ac - BAq+DA = F. Which is to be resolved after the Rules for extraction of those Rootes.

Question 27. Having the difference of the Cubes, and the difference of the Numbers, *viz.* X and 'X to finde out the Numbers themselves.

By the 15 *Question*, X = √c'X+Ec: - E and X+E = √c'X+Ec: and Xc+3XqE+3XEq+Ec = 'X+Ec and Ec+ $\frac{3X}{2}$ *Eq+ $\frac{3Xq}{2}$ +E = $\frac{'X - Xc}{2}$

Question 28. Having the Rectangle and the R

of two Numbers given, (that is, having P and $\frac{R}{S}$)

finde A, E, Z, X, Z, and X,.

First I finde out A and E by the 7 *Quest.* S =

and AqS = RP and $A = \sqrt{\frac{RP.}{S}}$ $E = \sqrt{\frac{RS.}{R}}$

A		$\sqrt{\frac{PR}{S}}$
E		$\sqrt{\frac{PS}{R}}$
Z		$\sqrt{\frac{PR}{8}} + \sqrt{\frac{PS}{R}}$
X	is	$\sqrt{\frac{PR}{S}} - \sqrt{\frac{PS}{R}}$
Aq		$\frac{PR}{S}$
Eq		$\frac{PS}{R}$
Z,		$\frac{PRq + PSq}{RS}$
X,		$\frac{PRq - PSq}{RS}$

Question 2

Question 29. Having the Rectangle and the sum the squares of two Numbers &c. or having P and ,to finde out A, E, Z X, $\frac{R}{S}$, X,

First by the 6 *Question* I finde A and E thus.

$$, = \frac{Aqq + Pq}{Aq} \text{ and } Z{,}Aq : Aqq = Pq.$$

Wherefore

$$\begin{array}{l|c|l}
A & & \sqrt{:}\ \frac{Z,}{2} + \sqrt{\frac{Zq - 4Pq}{4}} \\
E & & \sqrt{:}\ \frac{Z,}{2} - \sqrt{\frac{Z,q - 4Pq}{4}} \\
Z & & \sqrt{:}\ Z,+2P: \\
X & & \sqrt{:}\ Z,\cdot 2P: \\
\frac{R}{S} & & \frac{\sqrt{:}\ 2Z,Rq - \sqrt{4ZqRqq - 16PqRqq}:}{\sqrt{:}\ 2Z,+\sqrt{Z,q - 16Pq}:} \\
Aq & is & \sqrt{:}\ \frac{Z,q - 2Pq}{2} \pm \sqrt{\frac{Z,qq - 4Z,Pq}{4}} \\
Eq & & \sqrt{\frac{Z,q \cdot 2Pq}{2}} \cdot \sqrt{\frac{Z,qq - 4Z,Pq}{4}} \\
X, & & \sqrt{Z,q \cdot 4Pq}:
\end{array}$$

Question 30.

Question 30. Having the Rectangle, and the di-ference of the squares of two Numbers to finde o the Numbers themselves, that is, Having P and X g ven to finde **A, E, Z, X,** $\frac{R}{S}$, **Z.**

First by *Question* 6. $X = \frac{Aqq - Pq}{Aq}$ and XA $= Aqq - Pq.$ and $Aqq - X Aq = Pq.$
Therefore

A		$\sqrt{:}\sqrt{\frac{Xq+4Pq}{4}}: + \frac{X}{2}$
E		$\sqrt{:}\sqrt{\frac{Xq+4Pq}{4}}: - \frac{X}{2}$
Z		$\sqrt{:}\sqrt{Xq+4Pq}: +2P$
X	is	$\sqrt{:}\sqrt{Xq+4Pq}: - 2P.$
$\frac{R}{S}$		$\frac{\sqrt{:}\sqrt{XqRqq+4PqRqq}: - 2XR}{\sqrt{:}\sqrt{Xq+4Pq}: +2X.}$
Aq		$\sqrt{:}\sqrt{\frac{Xqq+4XqPq}{4}}: + \frac{Xq+2P}{2}$
E		$\sqrt{:}\sqrt{\frac{Xqq+4XqPq}{4}}: - \frac{Xq+2P}{2}$
Z		$\sqrt{Xq+4Pq.}$

Question 3

Question 31. Having P and Z given to finde out first Numbers.

the 14 *Question* P $=\sqrt{c}$ ZAc - Acc and ZAc - c $=$ Pc. and

$$\frac{Z}{2} \pm \sqrt{\frac{Zq-4Pc}{4}} := \begin{matrix} A \\ E. \end{matrix}$$

Question 32. Having P and X given to finde A E *per* 14. *Question*.

$= \sqrt{}$ X Ec+Ecc and Pc $=$ X Ec+Ecc.

Wherefore.

$$\sqrt{}: \frac{Xq+4Pc}{4} : \pm \frac{X}{2} = \begin{matrix} A \\ E. \end{matrix}$$

Question 33.

Question 33. Having the Proportion of Numbers, and the summes of the squares, viz and Z to finde A, E, Z X, P, X.

First, *per Question* 8. $Z = \frac{RqAq+SqAq}{Rq}$

and $Z\,Rq = RqAq+SqAq$ and $A - \sqrt{\frac{ZRq}{Rq+Sq}}$

$$E = \frac{\sqrt{ZSq}}{Rq+Sq}$$

A	$\sqrt{\frac{ZRq}{Rq+Sq}}$
E	$\sqrt{\frac{ZSq}{Rq+Sq}}$
Z	$\sqrt{\frac{ZRq}{Rq+Sq}} + \sqrt{\frac{ZSq}{Rq+Sq}}$
X	$\sqrt{\frac{Z,Rq}{Rq+Sq}} - \sqrt{\frac{Z,Sq}{Rq+Sq}}$
P	$\sqrt{\frac{ZqRqSq}{Rqq+2RqSq+Sqq}}$
Aq	$\frac{Z,Rq}{Rq+Sq}$
Eq	$\frac{Z,Sq}{Rq+Sq}$
X	$\frac{Z,Rq - Z,Sq}{Rq+Sq}$.

Questi

Question 34. Having the proportion of two [nu]mbers and the difference of their squares, *viz.* $\frac{R}{S}$ X, to seeke A, E, Z, X, P, Z,.

[F]irst 8 *Question.* It is found by supposition, that

$= \frac{RqAq - SqAq}{Rq}$ therefore $X,Rq = RqAq - SqAq$

[th]erefore $A = \frac{X,Rq}{Rq - Sq}$

$E = \frac{X,Sq}{Rq - Sq}.$

A	$\sqrt{\frac{X,Rq}{Rq - Sq}}$
E	$\sqrt{\frac{X,Sq}{Rq - Sq}}$
Z	$\sqrt{\frac{X,Rq}{Rq - Sq}} + \sqrt{\frac{X,Sq}{Rq - Sq}}$
X	$\sqrt{\frac{X,Rq}{Rq - Sq}} - \sqrt{\frac{X,Sq}{Rq - Sq}}.$
P	$\sqrt{\frac{X,qRqSq}{Rqq - 2RqSq + Sqq}}.$
Aq	$\frac{X,Rq}{Rq - Sq}$
Eq	$\frac{X,Sq}{Rq - Sq}$
Z,	$\frac{X,Rq + X,Sq}{Rq - Sq}.$

Qu. 35.

Queſtion 35. Having the proportion and ſum of the Cubes of two Numbers given, *viz.* $\frac{R}{S}$ and to finde A and E.

By the 14 *Queſtion.* $S = \sqrt{c}\,\frac{\cdot ZRc - RcAc}{Ac}$

and $ScAc = \cdot ZRc - RcAc$ and $RcAc + ScAc = \cdot Z$

Wherefore $A = \sqrt{c}\,\frac{\cdot ZRc}{Rc+Sc}$ $E = \sqrt{c}\,\frac{\cdot ZSc}{Rc+Sc}$

The like worke will ariſe if $\cdot X$ and $\frac{R}{S}$ were ven for

$$A = \sqrt{c}\,\frac{\cdot XRc}{Rc - Sc.} \quad E = \sqrt{c}\,\frac{\cdot XSc}{Rc - Sc.}$$

Queſtion

Question 36. Having the summe and difference of the squares of two Numbers given to finde the Numbers themselves, *viz.* having Z, and X, to finde A, E, Z, X, P, $\frac{R}{S}$.

First I seeke A and E by the 10 *Question*, where $X, = 2Aq - Z$ therefore $Z, + X, = 2Aq$, and $A = \sqrt{\frac{Z+X,}{2}}$ and $E = \sqrt{\frac{Z, - X,}{2}}$

A	$\sqrt{\frac{Z+X,}{2}}$
E	$\sqrt{\frac{Z, - X,}{2}}$
Z	$\sqrt{\frac{Z, + X}{2}} + \sqrt{\frac{Z, - X,}{2}}$
X	$\sqrt{\frac{Z+X,}{2}} - \sqrt{\frac{Z, - X,}{2}}$
P	$\sqrt{\frac{Z,q - X,q}{4}}$
$\frac{R}{S}$	$\sqrt{\frac{Z,Rq - X,Rq}{Z+X,}}$

Question 37. Having $Z,$ and $'Z$ given to finde out the Numbers, *viz.* A and E. By 14. $Z, = Aq + \sqrt{c}$ $'Zq - 2'ZAc + Acc$ and $Z,q - 2Z,A + Aqq = 'Zq - 2ZAc + Acc.$ and $2ZAc - Acc - 2Z,A + Aqq = 'Z - Zq$ and $- Acc + Aqq + 2ZAc - 2Z,A = 'Z - Z,q.$

Question 38. Having the summe of two Numbers and the proportion, the difference beareth to the sum of the squares, *viz.* R. S. to finde these two Numbers *per Question* 2. $R.S :: 2A - Z. Zq - 2ZA + 2Aq.$

Wherefore $2SA - ZS = ZqR - 2ZRA + 2RAq.$

And $2SA + 2ZRq = ZqR + ZS.$

$$\text{and } \frac{2S + 2ZR}{2R} A - Aq = \frac{ZqR + ZS}{2R}$$

The which varieties are infinite, as taking what proportion you please.

Question 39. Having the proportion of two Numbers R. S. and the proportion of their summe, and summe of squares are K. T. to finde out the Numbers.

Let the greater Number be A then by the 9. *Quest.*

$$Z = \frac{RA + SA}{R} \text{ and } Z, = \frac{RqAq + SqAq}{Rq.}$$

$$\text{Wherefore } \frac{RA + SA}{R} \cdot \frac{RqAq + SqAq}{Rq} :: K.T.$$

$$\text{Wherefore } \frac{RTA + STA}{R} = \frac{RqKAq + SqKAq}{Rq}$$

and $RqTA + SRTA = RqKAq + SqKAq.$

$$\text{and } \frac{RqT + SRT}{RqK + SqK} = A.$$

Quest. 4

Question 40. Having the proportion of two Numbers R. S. and the proportion of their squares to be K. T. to finde these two Numbers: Let the greater be A then by the 8 *Question* K. T : : Aq. $\frac{SqAq}{Rq}$

then $AqT = \frac{SqKAq}{Rq}$ and $RqTAq = SqRAq$.

Therefore I conclude that I cannot finde the Numbers by the premised *Data*, for there was nothing given but the bare Proportions, of two Numbers the proportions of their squares being only Rq. Sq. for RqT = SqK and Rq. Sq : : K. T. and so I conclude an impossibility, it being to be satisfied by any Number in proportion of R. S.

Question 41. Having the proportion of two Numbers as R. S. and the proportion of the Rectangle to the summe of the Cubes to be as K. T. to finde A and E.

$$K.\ T : : \frac{SAq}{R} \quad \frac{RcAc+ScAc}{Rc}$$

$$\text{and } TSRAq = RcKAc+ScAc$$

$$\text{and } \frac{RST}{RcK+Sc} = A.$$

Question 42. The alteration of Questions and varieties are infinite; I am very certaine every Rule delivered in this part hath been practised in some of these questions before recited, which was that I aymed at, by which a way is laid for the ready understanding of *Diophantus &c.* for some of these answer to 1, 2, 3, 4, 5, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42 of the first booke, 1, 2, 3, 4, 5 of the second, *&c.*

Out of the premises it appeares.

$$A = Z - E = X + E = \frac{P}{E} = \frac{RE}{S} = \sqrt{Z - Eq}:$$

$$= \sqrt{X + Eq} = \sqrt{c}{:}X + Ec = \frac{Z+X}{2} = \frac{Z}{2} + \sqrt{\frac{Zq - 4P}{4}}$$

$$= \frac{ZR}{R+S} = \frac{Z}{2} + \sqrt{\frac{Zq - 2Z + 2Z,}{4}} = \frac{Zq + X,}{2Z} =$$

$$\frac{Z}{2} + \sqrt{\frac{Zq - Zc + {}'Z}{4}} = \frac{Xq + 4P}{4} + \frac{X}{2} = \frac{RX}{R-S} =$$

$$\sqrt{\frac{2Z, - Xq}{4}}: + \frac{X}{2} = \frac{X, + Xq}{2X} = \sqrt{\frac{PR}{S}} = \sqrt{:}\frac{Z,}{2}$$

$$+ \sqrt{\frac{Zq - 4Pq}{4}} = \sqrt{:}\sqrt{\frac{X,q + 4Pq}{4}}: + \frac{X,}{2} = \sqrt{:}\frac{{}'Xq + 3P}{4}$$

$$+ \frac{X,}{2} = \sqrt{\frac{Z,Rq}{Rq+Sq}} = \frac{X,Rq}{Rq - Sq} = \frac{{}'XRc}{Rc - Sc}$$

$$= \sqrt{\frac{Z, + X,}{2}}$$ &c. and so of any other.

Questi

Questions concerning meane proportionalls; and first of three meane proportionalls.

The Propositions following resolve all Questions about 3 meane proportionalls; having 2 quantities or the proportion, and one quantity of any of those specified Chap. IX. Sect. 1.

$\Psi = A+M+E$. 123
$\Sigma = A+M$. 1. 2
$\Theta = M+E$. 1 2
$O = M-E$. 2 3
$Z = A+E$. 1 3
$X = A-E$. 1 3
$V = Aq+Mq$. 1. 2
$\Lambda = Aq-Mq$ 1 2
$\Omega = Mq+Eq$ 2 3
$\mho = Mq-Eq$ 2 3
$Z, = Aq+Eq$ 1 3
$X, = Aq-Eq$ 1 3
$P = A \times E$ 1 2
$\Phi = M \times E$ 2 3
$Æ = A+E$ 1 3
$\mathcal{N} = aq+mq+eq$. 123

Prop. 1. Having **M** and X to finde **A** and E, I put α for the first, then $\alpha - X = E$. Therefore α. M. $\alpha - X$ ∺ and $Mq = \alpha q - X\alpha$ and $\alpha = \sqrt{\frac{Xq+4Mq}{4}} :+ \frac{X}{2}$.

Prop. 2. Having M and Z to finde A and E, let α be the first Number, then $\alpha.$ M. Z. $- \alpha \div$ Mq $=$ Zα $- \alpha$q and $\frac{Z}{2} \pm \sqrt{\frac{Zq + 4Mq}{4}} = \frac{A}{E.}$

Prop. 3. Having A and Θ to finde M and E let α be put for M, E $= \Theta - \alpha$ and A. $\alpha . \Theta . \alpha \div . \alpha$q $=$ ΘA+Aα. Aα +αq $=$ ΘA.

Prop. 4 Having E and Ξ to finde A and M Let α $=$ M and $\Xi - \alpha$ α E $\div$ wherefore αq $= \Xi$ E $- \alpha$ E and α E+αq $= \Xi$ E.

Prop. 5. Having X and Θ to finde A M E $\div$ Let α be put for E, then $\Theta - \alpha$ is M. & α +X$=$A. therefore α +X. $\Theta - \alpha . \alpha \div$ and Θq $-$ 2$\Theta\alpha$ +αq $=$ αq+Xα and

$$\Theta q = X\alpha + 2\Theta\alpha \text{ and } \alpha = \frac{\Theta q}{2\Theta + X.}$$

Prop. 6. Having Ψ and ℳ given to find A,M,E: Let α be put for M, therefore A+E $= \Psi - \alpha$ therefore

$$\alpha q = \Psi q - 2\Psi\alpha = ℳ \text{ and } \alpha = \frac{ℳ}{\Psi q - 2\Psi} \quad \frac{ℳ - \Psi q}{2\Psi}$$

Prop. 7. Having the proportion of Aq to ℳ R S to finde A. M. E. Let α be put for A, then let S be put for E, and α S $=$ Q. M. and αq. Sα +Sq : : R. S and Sαq $=$ α RS+RSq. Sαq $-$ α RS $=$ RSq. and αq $-$ α R $=$ RS, or contrarily, S. α S+αq : : R. S. then αq $=$ α RS+Rαq and α S+Sq $= \frac{Sq}{R.}$

Prop.

Prop. 8. Having X and Ω given to finde A, M, E $\because$
Let α be put for Z, then $\alpha + X = 2A$ and $\alpha - X = 2E$
and $A + E = \frac{\alpha + X}{2} + \frac{\alpha - X}{2}$ and $\frac{\alpha q - Xq}{4} = Q{:}\, M$.
and $\frac{\alpha q - Xq}{4} + \frac{\alpha q - 2\alpha X + Xq}{4} = \Omega$ and $\frac{2\alpha q - 2\alpha X}{4} = 4\Omega$ and $\alpha q - X\alpha = 2\Omega$.

Prop. 9. Having X and ℳ given to find A M, E $\because$
Let α be put for Z therefore $\alpha + X = 2A$ & $\alpha - X = E$
wherefore $\frac{\alpha + X}{2} + \frac{\alpha - X}{2} = A + E$ and $\frac{\alpha q - Xq}{4} = Mq$
and $\frac{2\alpha q + 2Xq}{4} = Z$ and $\frac{\alpha q - Xq}{4} + \frac{2\alpha q + 2Xq}{4} = ℳ$
and $3\alpha q = 4ℳ - Xq$ and $\alpha = \sqrt{\frac{4ℳ - Xq}{3}}$

Prop. 10. Let A.M.N.E $\because$ and having $A - E = Z$ and $M - N = X$ given to finde out the foure Prop.
Let α be put for N, then $\alpha + X = M$:

therefore $\frac{\alpha q + 2\alpha X + Xq}{\alpha} = A$ and $\frac{\alpha q}{\alpha + X} = E$

and $\frac{3\alpha qX + 3\alpha Xq + Xc}{\alpha q + \alpha X} = Z$

and $\alpha qZ + \alpha ZX - 3\alpha qX - 3\alpha Xq = Xc$

and $\frac{ZX - 3Xq}{Z - 3X} \ast \alpha + \alpha q = \frac{Xc}{Z - 3X}$

or $\underline{Z - X} \ast \alpha + \alpha q = \frac{Xc}{Z - 3X}$.

These and such like may be put and resolved infinit-

Chap. XI.

Containing many Questions of severall subjects.

PRop. 1. To finde a Number that multiplyed by B (6) and the Product added to C 8, doth make D (48. Let the Number sought be A, then

$B \times A + C = D$ and $A = \frac{D-C}{B}$

which is equall to $6\frac{2}{3}$.

Prop. 2. To divide B (100) into A and E, that $\frac{A}{3} + \frac{E}{5} = D.$ 30.

Let $\alpha = A$ and $B - \alpha = E$, therefore $\frac{\alpha}{3} + \frac{B-\alpha}{5} = D$

and $5\alpha + 3B - 3\alpha = 15\,D$.

and $\alpha = \frac{15D - 3B}{2} = 75$.

Prop. 3. To finde A and E, that $A = E + B$ (4) and $Aq = Eq + D$ (32) Let α be put for A, then $\alpha - B = E$, and $\alpha\,q = \alpha\,q - 2B\alpha + D$ and $\alpha = \frac{Bq + D}{2B}$

and $\alpha = 6 = A$ and $E = 2$.

Prop. 4. To finde A and E, that $Aq - Eq = B$ (6 and $A - E = X$ (5) Let α be put for A, then $\alpha - X = E$

The

Then $\alpha q + \alpha q - 2\alpha X + Xq = B.$

and $\alpha q - X\alpha = \dfrac{B - Xq}{2.}$

Prop. 5. To finde a Number that being joyned with B (18) and taken from C (100) the summe and remaine shall be as R. S. (1. 3) Let it be A. Then B+A. C - A :: R. S.

Then $RC\text{-}RA = BS + SA$ and it is $A = \dfrac{RC\text{-}BS}{R+S} = 11\frac{1}{2}.$

Prop. 6. To divide B (30) into A, M, N, E ∺ that A may be = to C (2)

Put α for M. then $\dfrac{\alpha q}{c} = N$ &c. $\dfrac{\alpha c}{Cq} = E.$

Therefore $c.\ \alpha.\ \dfrac{\alpha q}{c}\ \dfrac{\alpha c}{cq}$ ∺ = B By Addition.

$\dfrac{Cc + Cq\alpha + C\alpha q + \alpha c}{Cq} = B$ and $\alpha c + C\alpha q + Cq\alpha = BCq - Cc.$

Now $BCq - Cc = 112 = Ac + CAq + CqA.$

The Canon will be made thus.

Ac	+ CAq	+ Cq:A . . . . . .
3 Aq:E	+ C2A:E	+ Cq:E
3 A:Eq	+ C:Eq	:
:Ec	:	

$$
\begin{array}{lrr}
 & : & \\
 & \vdots & \\
 & 112 & (4 = a. \\
 & : & \\
\hline
Cq\ldots & 4 & \\
C\ldots & 2 & \\
\hline
Dr\ldots & 6 & \\
\hline
CqA & +\ 16 & \\
CAq & +\ 32 & \\
Ac & +\ 64 & \\
\hline
 & 112 & \\
\hline
\end{array}
$$

Therefore 2. 4. 8. 16 $\div\!\!\div$ and $2+4+8+16 = 30$

Prop. 7. To finde a Number from which if you take B (3) and to $\frac{1}{B}$ thereof adde C (7:) The summe being drawne in B, and from the product take D (18) the remaine shall be equall to F (21:) Let the Number sought be A, and according to the Question, A - B the $\frac{1}{B}$ of it is $\frac{A-B}{B}$ adde C it is $\frac{A-B+BC}{B}$ $\times B = A - B + BC$ & $A+BC-B-D = F$. $A = F-BC+B+D$ & $A = 2$:.

Prop. 8. To finde a Number from the triple, whereof if I take B (30) and from the double of that if I take C (140,) and if I draw the rest in D (4) and from the product take F (100) there remains nothing.

Let A be put for the sought Quantity, then 3A - B and $6A - 2B - C$ and $6DA - 2BD - DC - F = 0$ therefore $6DA = 2BD + DC$ and $A = \frac{2BD+DC}{6DA} =$

9. Two

9. Two Numbers are sought in triple proportion, nd the lesse taken from the greater leaveth a Number quall to the Quotient of the greater divided by the esse. Let the greater be A and the proportion $\frac{R}{S}$.

hen $\frac{SA}{R} =$ the lesser and $\frac{RA - SA}{R} = \frac{SA}{RA}$ and $RA -$ $SA = S$, $A = \frac{S}{R - S}$ $A = \frac{3}{2}$ $E = \frac{9}{2}$.

10. A Number is sought, from which take $\frac{R}{S} \frac{(2)}{(5)}$ t makes the Number as much under B (100) as it was at first above B. Put A for the Number sought. Then $A - \frac{R}{S} = \frac{SA - RA}{S}$ and $B - \frac{SA - R}{S} = A - B$ nd $2B = \frac{2SA - RA}{S}$ and $2BS = 2SA - RA$ nd $A = \frac{2BS}{2S - R}$.

11. The resolutions of the Questions, Chap. IX. Sect. 3. Of the first, $BA + C = DA$ $C = DA - BA$ nd $A = \frac{C}{D - B}$.

12. Of the second, for the greater I put A the esser $B - A$, then $CA = BD - DA$ and $CA + DA =$ BD and $A = \frac{BD}{C + D}$.

13. Of

13. Of the third, for the one put A, then B - A i[s] the other, and $CA+DB-DA = F$ and $CA-DA =$ $F-DB$ and $A = \frac{F-DB}{C-D.}$

14. Of the fourth, let the one be A, the other i[s] B - A, then $\frac{A}{C} + \frac{B-A}{D} = F$ and $DA+BC-CA =$ DCF and $A = \frac{DCF-BC}{D-C.}$

15. Of the fifth, let the greater be A, then A - X is the other: therefore $BA+BA-BX = C$. and $A = \frac{C+BX}{2B.}$

16. Of the sixth, if you put the lesser Divisor A, th[e] greater is A+C; & $\frac{B}{A} - \frac{B}{A+C} = D$ and $BA+BC-B[A]$ $= DAq+DA+DC$ and $BC-DC = DAq-DA$. and $Aq - 1A = \frac{BC-DC}{D.}$

17. Of the seaventh, for the sought Mag. put A, then $BA-D = CA-F$, then $F-D = CA-BA$. and $A = \frac{F-D}{C-B.}$

18. To finde two Numbers in that proportion that the $\frac{1}{2}$ of the second more by B (2) added to th[e] former, is 9 times as much as the remainder of the se-cond: But the third part of the first more by C (3) ad-ded to the second, is triple to the Remainer of th[e] first

ſt: I put for the firſt E, and for the ſecond 2A:
en $A+B+E = (A-B \times 9) = 9A-9B$, and $9A-9B-$
$B = (E) = 8A-10B$. and $\frac{8A-10B}{3C}+C+$

$A = \frac{14A-10B+3C}{3} = \frac{16A-20B-3C}{8} \times \frac{8}{1}$

d $14A-10B+3C = 48A-60B-9C$ and $60B+9C-$
$B+3C = 48A-14A$ and $50B+12C = 34A$.

d $A = \frac{50B+12C}{34} = 4$ and $2A = 8$.

Then $64-20 = 2E = 24 = 12$.

19. Two men had ſeverally certaine ſummes of
ownes, the proportion was as $\frac{R}{S}\frac{(4)}{(1)}$ the ſumme
at both had wanted of B (100) but the ſumme
ubled and made leſſe by (D) (29) there was twice
much above B, as there wanted at firſt of B, I de-
and what either man had. Firſt I finde Z the ſumme,
d for the want which the ſumme was ſhort of B, I
t E, Then $E = B-Z$ and $Z = B-E$, but $2Z-D$
$= B+2E$, that is $2Z-D = B+2B-2Z$ and $4Z =$
$B+D$ and $Z = \frac{2B+D}{4}$ that is (80.) Now having Z
d $\frac{R}{S}$ by the 18 *Queſt.* of this Chap. then $A = \frac{ZR}{R+S}$
$= \frac{ZS}{R+S}$ Therefore A = 64. E. 10.

20. A Merchant hath a hogſhead of wine of B (360)
Gallons

Gallons, he poured out three severall times a certai[n] number of Gallons, and filled them up with wate[r] at the last there remained D ($108\frac{1}{3}$) of wine, wh[at] was poured out at each time? Betwixt B and D fin[d] two meane proportionals, thus B, $\sqrt{c}BqD$, $\sqrt{c}BD$ D: Then A = B − $\sqrt{c}BqD$:

The same question may be wrought by prosecutin[g] the tenor of the Question thus.

The first draught, B − A.

The second draught, Bm − Am. resteth in the Hog[s]head of mixt $\dfrac{Bq - 2BA + Aq}{B}$

found thus, B. B − A :: B − A. $\dfrac{Bq - 2BA + Aq}{B}$

The third draught is Bm − Am. rests of wi[ne] $\dfrac{Bc - 3BqA + 3BAq - Ac}{Bq}$ Found by this Analogy.

A B − A :: $\dfrac{Bq - 2BA + Aq}{B}$. $\dfrac{Bc - 3BqA + 3BAq - A[c]}{Bq}$

Wherefore $\dfrac{Bc - 3BqA + 3BAq - Ac}{Bq}$ = D and B[c −]

3BqA + 3BAq − Ac = BqD.

And by Reduction Ac − 3BAq + 3BqA = BqD

in Numbers. Ac − 1080Aq + 388800A = 19656[...]

A = 60.

```
   .  .  .
 1965600 0 (60
  .  :.:
 388800-   +3Bq
  1080--    3B
 398003    A
 2332800   +3BqA
 -38880    -3BAq
   216     +Ac
+2354400.
 1965600   Ab.
  000000   Resid.
     .
```

For the invention of as many meane Proportionals you please, (which may concerne the former part this Question) you may take notice they may all be found by the extracting of the sq. and cube Roots, but because in the fourth power it is,

	A	=	A	
$\sqrt{}$qq.	AcE	= $\sqrt{}$q	AcE	All to be extracted by the square Root.
$\sqrt{}$qq.	AqEq	= $\sqrt{}$q	AE	
$\sqrt{}$qq.	AEqq	= $\sqrt{}$q	AEc	
	E		E	

A

	A		A	
$\sqrt{qc}$	AqqE	$\sqrt{q}$	AqcE	All to be extracted by the sq. and cube Roots, &c.
$\sqrt{qc}$	AcEq	$\sqrt{c}$	AqE	
$\sqrt{qc}$	AqEc	$\sqrt{c}$	AEq	
$\sqrt{qc}$	AEqq	$\sqrt{q}$	AEqc	
	E		E	

21. To divide Z (100) into A and E, that $\frac{A}{4}$ may exceed $\frac{E}{6}$ by C. (20) Let A = A. E = Z − A. then $\frac{A}{4} = \frac{Z-A}{6} + C$ and $6A = 4Z - 4A + 24C$ and $10A = 4Z + 24C$ and $A = \frac{4Z+24C}{10} = 88.$

22. To divide Z into A, E and O; That $A+E = 3O$ and $O+E = 4A$. Therefore $4O = Z$ and $O = \frac{Z}{4}$ and $5A = Z$. $A = \frac{Z}{5}$.

23. To finde A, E, O, that $A+E+\frac{O}{2} = Z$. $E+O+\frac{A}{3} = Z$ and $A+O+\frac{E}{4} = Z$.

Wherefore (1) $2A+2E+O = 2Z$
(2) $3E+3O+A = 3Z$

(3) $4A+4O+E = 4Z$

By substracting the (3) from (1, 2) $4E-A = Z$

and $E = \frac{Z+A}{4}$

By sub. the (2 e, 1, 3) $5A+2O = 3Z$.

and $O = \frac{3Z-5A}{2}$

By sub. the 3 from (1 and 2 in 2) $7E+3O = 4Z$, therefore it is $7Z+7A+18Z-15A = 16Z$

and $\frac{9Z}{23} = A$ &c.

24. To finde three Numbers, *viz.* A, E, O, with this condition that $A+\frac{2}{3}E+\frac{2}{3}O = Z$ (100.) $E+\frac{3}{4}A+\frac{3}{4}O = Z$. and $O+\frac{4}{5}A+\frac{4}{5}E = Z$.

$$\left.\begin{array}{ll}(1) & 3A+2E+2O = 3Z \\ (2) & 3A+4E+3O = 4Z \\ (3) & 4A+4E+5O = 5Z\end{array}\right\}\text{Reduced into Integers.}$$

By sub. (2, e, 3) it is $A+2O = Z$. and $O = \frac{Z-A}{2}$

By sub. (3, e, 1 and 2) it is $2A+2E = 2Z$ and $E = Z-A$.

By sub. (1, e, 2) it is $2E+1O = Z$, therefore $4Z-4A+Z-A = 2Z$.

and $\frac{3Z}{5} = A = 60$. E 40. O. 20.

Prop. 25. To finde foure Numbers, *viz.* A, E, I, O, that $A+\frac{1}{2}E+\frac{1}{2}I+\frac{1}{2}O = B$, and $E+\frac{1}{3}A+\frac{1}{3}I+\frac{1}{3}O = C$, and $I+\frac{1}{4}A+\frac{1}{4}E+\frac{1}{4}O = D$,

and $O+\frac{A}{6}+\frac{E}{6}+\frac{I}{6} = D$.

$$\left.\begin{array}{ll}(1) & 2A+E+I+O = 2B\\ (2) & A+3E+I+O = 3C\\ (3) & A+E+4I+O = 4D\\ (4) & A+E+I+6O = 6D\end{array}\right\}\text{Reduced to Integers.}$$

Sub 1, from 3 and 4 it is $E+4I+6O = (10D-2B) = F$
1, from 2 —— —— $2E-A = (3C-2B) = G$
1, from 3 —— —— $3I-A = (4D-2B) = H$
1, from 4 —— —— $4O-A = (6D-2B) = K$

Wherefore $E = \frac{G+A}{2}$. $I = \frac{H+A}{3}$. $O = \frac{K+A}{5}$.

Wherefore $\frac{G+A}{2}+\frac{4H+4A}{3}+\frac{6K+6A}{5} = F$, and by Reduction, $15G+15A+40H+36K+36A = 30F$, and $A = \frac{30F-15G-40H-36K}{91}$.

Wherefore A6. E4. I8. O10.

Prop. 26. To divide Z into A and E, that $A*E = B+\sqrt{q}C$. $E = Z-A$, and $ZA-Aq = B+\sqrt{q}C$, and $ZA-Aq-B = \sqrt{q}C$. Wherefore $ZqAq-2ZAc-2ZBA+Aqq+2BAq+Bq = C$.

And $Aqq-2ZAc+\underline{Zq+2B}*Aq-2ZBA = C-Bq$.

Prop. 27.

Prop. 27. There is a ſquare, whoſe ſide drawne into the difference of the ſide and diameter produceth B. It is demanded to know the ſide and Diameter?

Let A be the ſide, then $\frac{B}{A}$ = Diff. of the ſide and Diameter, and $\frac{B}{A}$ + A = Diam. = $\frac{B+Aq}{A}$ and by the (47 e 1) $\frac{Bq+2BAq+Aqq}{Aq}$ = 2Aq.

and Bq+2BAq+Aqq = 2Aqq. and Aqq − 2BAq = Bq, and A = √: √q2Bq: +B. and Aq = √q2Bq: +B. Wherefore Q: Diam: = √q8Bq: +2B. and the Diam. = √q: √q8Bq: +2B. and the Diff: of L: and D: is √q: √q8Bq: +2B. − √q: √q2Bq: +B, which multiplyed together, at the laſt leaveth B.

28. To cut Z into A and E, that Eq×A ſhall be to X as R. S. E = Z − A. Eq = Zq − 2ZA+Aq; ×A = ZqA − 2ZAq+Ac, X : : R. S. that is ZqA − 2ZAq+Ac. 2A − Z : : R. S. and 2RA − ZR = ZqSA − 2ZSAq +SAc. and Ac − 2ZAq+ $\frac{2R}{S}$ +Zq×A = $\frac{ZR}{S}$.

Soli Deo gloria.

A CANON

Of the ſquares and cubes of all Numbers under 1000, of the ſquared ſquares, and under 500, and of the ſquare cubes, and cubed cubes under 250.

Very uſefull for extracting all ſorts of Rootes, whether ſimple or adfected:

As alſo in all Military buſineſſe, whether for the ordering of Battalions, Gunnery, *&c.* and meaſuring Solids, *&c.*

L	Aq	Ac	Aqq
1	1	1	1
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625
6	36	216	1296
7	49	343	2401
8	64	512	4096
9	81	729	6561
10	100	1000	10000
11	121	1331	14641
12	144	1728	20736
13	169	2197	28561
14	196	2744	38416
15	225	3375	50625
16	256	4096	65536
17	289	4913	83521
18	324	5832	104976
19	361	6859	130321
20	400	8000	160000
21	441	9261	194481
22	484	10648	234256
23	529	12167	279841
24	576	13824	331776
25	625	15625	390625

Aqc	Acc
1	1
32	64
243	729
1024	4096
3125	15625
7776	46656
16807	117649
32768	262144
59049	531441
100000	1000000
161051	1771561
248832	2985984
371293	4526809
537824	7529536
759375	11390625
1048576	16777216
1419857	24137569
1889568	34012224
2476099	47045881
3200000	64000000
4084101	85766121
5153632	113379904
6436343	148035889
7962624	191102976
9765625	244140625

26	676	17576	456976
27	729	19683	531441
28	784	21952	614656
29	841	24389	707281
30	900	27000	810000
31	961	29791	923521
32	1024	32768	1048576
33	1089	35937	1185921
34	1156	39304	1336336
35	1225	42875	1500625
36	1296	46656	1679616
37	1369	50653	1874161
38	1444	54872	2086136
39	1521	59319	2313441
40	1600	64000	2560000
41	1681	68921	2825761
42	1764	74088	3111696
43	1849	79507	3418801
44	1936	85184	3748096
45	2025	91125	4100625
46	2116	97336	4477456
47	2209	103823	4879681
48	2304	110592	5308416
49	2401	117649	5764801
50	2500	125000	6250000

11881376	308915776
14348907	387420489
17210368	481890304
20511149	594823321
24300000	729000000
28629151	887503681
33554432	1073741824
39135393	1291467969
45435424	1544804416
52521875	1838265625
60466176	2176782336
69343957	2564726409
79235168	3510936384
90224199	3018743761
102400000	4096000000
115856201	4750104241
130691232	5489031744
147008443	6321363049
164916224	7256313856
184528125	8303765625
205962976	9474296896
229345007	10779215329
254803968	12230590464
282475249	13841287201
312500000	15625000000

L	Aq	Ac	Aqq
51	2601	132651	6765201
52	2704	140608	7311616
53	2809	148877	7890481
54	2916	157464	8503056
55	3025	166375	9150625
56	3136	175616	9834496
57	3249	185193	10556001
58	3364	195112	11316496
59	3481	205379	12117361
60	3600	216000	12960000
61	3721	226981	13845841
62	3844	238328	14776336
63	3969	250047	15752961
64	4096	262144	16777216
65	4225	274625	17850625
66	4356	287496	18974736
67	4489	300763	20151121
68	4624	314432	21381376
69	4761	328509	22667121
70	4700	343000	24010000
71	5041	357911	25411681
72	5184	373248	26873856
73	5329	389017	28398241
74	5476	405224	29986576
75	5925	421875	21640625

Aqc	Acc
345025251	17596287801
380204032	19770609664
418195493	22164361129
459165024	24794911296
503284375	27680640625
550731776	30840979456
601692057	34809447249
656356768	38068692544
714924299	42180533641
777600000	46656000000
844596301	51520374361
916132832	56800235584
992436543	62523502209
1073741824	68719476736
1160290625	75418890625
1252332576	82653950016
1350125107	90458382169
1453933568	98867482624
1564031349	107918163081
1680700000	117649000000
1804229351	128100283921
1934917632	139314069504
2073071593	151334226289
2219306624	164206490176
2373046875	177978515625

76	5776	438976	33362176
77	5929	456533	35153041
78	6084	474552	37015056
79	6241	493039	38950081
80	6400	512000	40960000
81	6561	531441	43046721
82	6724	551368	45212176
83	6889	571787	47458321
84	7056	592704	49787136
85	7225	614125	52200625
86	7396	636056	54700816
87	7569	658503	57289761
88	7744	681472	59969536
89	7921	704969	62742241
90	8100	729000	65610000
91	8281	753571	68574961
92	8464	778688	71239296
93	8649	804357	74805201
94	8836	830584	78074896
95	9025	857375	81450625
96	9206	884736	84934656
97	9409	912673	88529281
98	9604	941192	92236816
99	9801	970299	96059601
100	10000	1000000	100000000

2535525376	192699928576
2706784157	208422380089
2887174368	224199600704
3077056399	243087455521
3276800000	262144000000
3486784401	282429536481
3707398432	304006671424
3939040643	326940373369
4182119424	351298031616
4437053125	377149515625
4704270176	404567235136
4984209207	433626201009
5277319168	464404086784
5584059449	496981290961
5904900000	531441000000
6240321451	567869252041
6554015232	620969401344
6956883693	646990183449
7339040224	689869781056
7737809375	735091890625
8153726976	782757789696
8587340257	832972004929
9039197968	885841400864
9509900499	941480149401
10000000000	1000000000000

L	Aq	Ac	Aqq
101	10201	1030301	104060401
102	10404	1061208	108243216
103	10609	1092727	112550881
104	10816	1124864	116985856
105	11025	1157625	121550625
106	11236	1191016	126247696
107	11449	1225043	131079601
108	11664	1259712	136048896
109	11881	1295029	141158161
110	12100	1331000	146410000
111	12321	1367631	151807041
112	12544	1404928	157351936
113	12769	1442897	163047361
114	12996	1481544	168896016
115	13225	1520875	174898625
116	13456	1560896	181089636
117	13689	1601613	187388721
118	13924	1643032	193877776
119	14161	1685159	200533921
120	14400	1728000	207360000
121	14641	1771561	214358881
122	14884	1815848	221533456
123	15129	1860867	228886641
124	15376	1906624	236421376
125	15625	1953125	244140625

Aqc	Acc
10510100501	1061520150601
11040808032	1126162419264
11592740743	1194052296529
12166529024	1265319018496
12762815625	1340095640625
13382255776	1418519112256
14025517307	1500730351849
14693280768	1586874322944
15386239549	1677100110841
16105100000	1771561000000
16850581551	1870414552161
17623416832	1973822685184
18424351793	2081951752609
19254145824	2194972623936
20113341875	2313034315625
21003416576	2436396322816
21924480357	2565164201769
22877577568	2699554153024
23863536599	2839760855281
24883200000	2985984000000
25937424601	3138428376721
27027081632	3297303959104
28153056843	3462825991689
29316250624	3635215077376
30517578125	3814697265625

126	15876	2000376	252047376
127	16129	2048383	260144641
128	16384	2097152	268435456
129	16641	2146689	276922881
130	16900	2197000	285610000
131	17161	2248091	294499921
132	17424	2299968	303595776
133	17689	2352637	312900721
134	17956	2406104	322417936
135	18225	2460375	332150625
136	18496	2515456	342102016
137	18769	2571353	352375361
138	19044	2628072	362673936
139	19321	2685619	373301041
140	19600	2744000	384160000
141	19881	2803221	395254161
142	20164	2863288	406586896
143	20449	2924207	418161601
144	20736	2985984	429981696
145	21025	3048625	442050625
146	21316	3112136	454371856
147	21609	3176523	466948881
148	21904	3241792	479785216
149	22201	3307949	492884401
150	22500	3375000	506250000

21657

Aqc	Acc
31757962376	4001504141376
33038369407	4195872914689
34359738368	4398046511104
35723051649	4608273662721
37129300000	4526809000000
38579489651	5053913144281
40074642432	5289852801024
41615795893	5534900853769
43203931064	5789326762712
44840334375	6053445140625
46525874176	6327518887936
48261724477	6611856253349
50049003168	6906762937184
51888844699	7212549413161
53782400000	7529536000000
55730836701	7858047975546
57735339232	8198418170944
59797108943	8550986578849
61917364224	8916100448256
64097340625	9294114390625
66338290976	9685390482496
68641485507	9890298369529
71008211968	10509215371264
73439775749	10942526586601
75937500000	11390625000000

L	Aq	Ac	Aqq
151	22801	3442951	519885601
152	23104	3511808	533794816
153	23409	3581577	547981281
154	23716	3652264	562448656
155	24025	3723875	577200625
156	24336	3796416	592240896
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163	26569	4330747	705911761
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167	27889	4657463	777796321
168	28224	4741632	794594176
169	28561	4826809	815730721
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185	34225	6331625	1171350625
186	34596	6434856	1196883216
187	34969	6539203	1222830961
188	35344	6644672	1249198336
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206	42436	8741816	1800814096
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208	43264	8998912	1871773696
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216	46656	10077696	2176782336
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232	53824	12487168	2897022976
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235	55225	12977875	3049800625
236	55696	13143256	3102044416
237	56169	13311053	3154956561
38	56644	13480272	3208542736
239	57121	13651919	3262808641
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241	58081	13997521	3373402561
242	58564	14172488	3429742096
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244	59536	14526784	3544535296
245	60025	14706125	3603000625
246	60516	14886936	3662186256
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255	55025	16581375	4228250625
256	65536	16777216	4294967296
257	66049	16974593	4392470401
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260	67600	17576000	4569760000
261	68121	17779581	4640470641
262	68644	17984728	5711998736
263	69169	18191447	4783505601
264	69696	18399744	4857532416
265	70225	18609625	4931550625
266	70756	18821096	5006411536
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268	71824	19248832	5158686976
269	72361	19465109	52361143[illegible]1
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283	80089	22665187	6414247921
284	81656	22906304	6505390336
285	81225	23149125	6597500625
286	81796	23393656	6690585616
287	82369	23639903	6784652161
288	82944	23887872	6879707136
289	83521	24137569	6975757441
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291	84681	24642171	7170871761
292	85264	24897088	7269949696
293	85849	25153757	7370059801
294	86436	25412184	7471182096
295	87025	25672375	7573350625
296	87616	25934336	7676563456
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304	92416	28094464	339	114921	38958219
305	93025	28372625	340	115600	39304000
306	93636	28652616	341	116281	39651821
307	94249	28934443	342	116964	40001688
308	94864	29218112	343	117649	40353607
309	95481	29503629	344	118336	40707584
310	96100	29791000	345	119025	41063625
311	96721	30080231	346	119716	41421736
312	97344	30271328	347	120409	41781923
313	97969	30664297	348	121104	42144192
314	98596	30659144	349	121801	42508549
315	99225	31255875	350	122500	42875000
316	99856	31554496	351	123201	43243551
317	100489	38855013	352	123904	43614208
318	101124	32157432	353	124609	43986977
319	101761	32461759	354	125316	44361864
320	102400	32768000	355	126025	44738875
321	103041	33076161	356	126736	45118016
322	103684	33386248	357	127449	45499293
323	109329	33698267	358	128164	45882712
324	104976	34012224	359	128881	46268279
325	105625	34328125	360	129600	46656000
326	106276	34645976	361	130321	47045881
327	106929	34965783	362	131044	47437928
328	107584	35287552	363	131769	47832147
329	108241	35611289	364	132496	48228544
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374	139876	52313624	409	167281	68417929
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376	141376	53157376	411	168921	69426531
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378	142884	54010152	413	170569	70444997
379	143641	54439929	414	171396	70957944
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395	156125	61629875	430	184900	79507000
396	156916	62099136	431	185761	80062991
397	157709	62570773	432	186624	80621568
398	158504	63044792	433	187489	81182737
399	159301	63521193	434	188356	81746504
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402	161604	64964808	437	190969	83453453
403	162409	65450827	438	191844	84027672
404	163216	65939264	439	192721	84604519
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444	197136	87528384	479	229441	109902239
445	198025	88121125	480	230400	110592000
446	198916	88716536	481	231361	111284641
417	199809	89314623	482	232324	111980168
448	200704	89915392	483	233289	112678587
449	201601	90518849	484	234256	113379904
450	202500	91125000	485	235225	114084125
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452	204304	92345408	487	237169	115501303
453	205209	92959677	488	238144	116214272
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457	208849	95443993	492	242064	119095488
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515	265225	136590875	550	302500	166375000
516	266256	137388096	551	303601	167284151
517	267289	138188413	552	304704	168196608
518	268324	138991832	553	305809	169112377
519	266361	139798359	554	306916	170031464
520	270400	140608000	555	308025	170953875
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525	275625	144703125	560	313600	175616000
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536	287296	153990656	571	326041	186169411
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606	367236	222545016	641	410881	263374721
607	368449	223648543	642	412164	264609288
608	369664	224755712	643	413449	265847707
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610	372100	226981000	645	416025	268336125
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654	427716	279726264	689	474721	327082769
655	429025	281011375	690	476100	328509000
656	430336	282300416	691	477481	329939371
657	436649	283593393	692	478864	331373888
658	432964	284890312	693	483249	332812557
659	434281	286191179	694	481633	334255384
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668	446224	298077632	703	494209	347428927
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673	452929	304821217	708	501264	354894912
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675	455625	307546875	710	504100	357911000
676	456976	308915776	711	505521	359425431
677	458329	310288733	712	506944	360944128
678	459684	311665752	713	508369	362467097
679	461041	313046839	714	509796	363994344
680	462400	314432000	715	511225	365525875
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682	465124	317214568	717	514089	368601813
683	466489	318611987	718	515524	370146232
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685	469225	321419125	720	518400	373248000

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737	543169	400315553	772	595984	460099648
738	544644	401947272	773	597529	461889917
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744	553536	411830784	779	606841	472729139
745	555025	413493625	780	608400	474552000
746	556516	415160936	781	609961	476279541
747	558009	416832723	782	611524	478211768
748	559504	418508992	783	613089	480048687
749	561001	420189741	784	614656	481890304
750	562500	421875000	685	616225	483736625
751	564001	423564751	786	617796	485587656
752	565504	425259008	787	619369	487443403
753	567009	426957777	788	620944	489303872
754	568516	428661064	789	622521	491169069
755	570025	430368875	790	624100	493039000

L	Aq	Ac	L	Aq	Ac
791	625681	494913671	826	682276	563559976
792	627264	496793088	827	683929	565609283
793	628849	498677257	828	685584	567663552
794	630436	500566184	829	687241	569722789
795	632025	502459875	830	688900	571787000
796	633616	504358336	831	690561	573856191
797	635209	506261573	832	692224	575930368
798	636804	508169592	833	693889	578009537
799	638401	510082399	834	695556	580093704
800	640000	512000000	835	697225	582182875
801	641601	513922402	836	698896	584277056
802	643204	515849608	837	700569	586376253
803	644809	517781627	838	702244	588480472
804	646416	519718464	839	703921	590589719
805	648025	521660125	840	705600	592704000
806	649636	523606616	841	707281	594823321
807	651249	525557943	842	708964	596947688
808	652864	527514112	843	710649	599077107
809	654481	529475129	844	712336	601211584
810	656100	531441000	845	714025	603351125
811	657721	533411731	846	715716	605495736
812	659344	535387328	847	717409	607645423
813	660969	537367797	848	719104	609800199
814	662596	539353144	849	720801	611960049
815	664225	541343375	850	722500	614125000
816	665856	543338496	851	724201	616295051
817	667489	545338513	852	725904	618470208
818	669124	547343432	853	727609	620650477
819	670761	549353259	854	729316	622835864
820	672400	551368000	855	731025	625026375
821	674041	553387661	856	732736	627222016
822	675684	555412248	857	734449	629422793
823	677329	557441767	858	736164	631628712
824	678976	559476224	859	737881	633839779
825	680625	561515625	860	739600	636056000

L	Aq	Ac	L	Aq	Ac
861	741321	638277381	896	802816	719323136
862	743044	640503928	897	804609	721734273
863	744769	642735647	898	806404	724150792
864	746496	644972544	899	808201	726572699
865	748225	647214625	900	810000	729000000
866	749956	649461896	901	811801	731432701
867	751689	651714363	902	813604	733870808
868	753424	653972032	903	815409	736314327
869	755161	656234929	904	817216	738763264
870	756900	658503000	905	819025	741217625
871	758641	660776311	906	820836	743677416
872	760384	663054848	907	822649	746142643
873	762876	665336617	908	824644	748613312
874	765625	667627624	909	826281	751089429
875	767376	669921875	910	828100	753571000
876	767376	672221376	911	829921	756058031
877	769129	674526133	912	831744	758550528
878	770884	676836152	913	833569	761048497
879	772641	679151439	914	835396	763551944
880	774400	681472000	915	837225	766060875
881	776161	683797841	916	839056	768575296
882	777924	686128968	917	840889	771095213
883	779689	688465387	918	842724	773620632
884	781456	690807104	919	844561	776151559
885	783225	693154125	920	846400	778688000
886	784996	695506456	921	848241	781229961
887	786769	697864103	922	850084	783777448
888	788544	700227072	923	851929	786330467
889	790321	702595369	924	853776	788889024
890	792100	704969000	925	855625	791453125
891	793881	707347971	926	857476	794022776
892	795664	709732288	927	859329	796597983
893	797449	712121957	928	861184	799178752
894	799236	714516984	929	863041	801765089
895	801025	716917375	930	864900	804357000

L	Aq	Ac	L	Aq	Ac
931	866761	806954491	966	933156	901428696
932	868624	809557568	967	935089	904231063
933	870489	812166237	968	937024	907039232
934	872356	814780504	969	938961	909853209
935	874225	817400375	970	940900	912673000
936	876096	820025856	971	942841	915498611
937	877969	822656953	972	944784	918330048
938	879844	825293672	973	946729	921167317
939	881721	827936019	974	948676	924010424
940	883600	830584000	975	950625	926859375
941	885481	833237621	976	952576	929714176
942	887364	835896888	977	954529	932574833
943	889249	838561807	978	956484	935441352
944	891136	841232384	979	958441	938313739
945	893025	843908625	980	960400	941192000
946	894916	846590536	981	962361	944076141
947	896809	849278123	982	964324	946966168
948	898704	851971392	983	966289	949862087
949	900601	854670349	984	968256	952763904
950	902500	857375000	985	970225	955671625
951	904401	860085351	986	972196	958585256
952	906304	862801408	987	974169	961504803
953	908209	865523177	988	976144	964430272
954	910116	868250664	989	978121	967361669
955	912025	870983875	990	980100	970299000
956	913936	873722816	991	982081	973242271
957	915849	876467493	992	984064	976191488
958	917764	879217912	993	986049	979146657
959	919681	881974079	994	988036	982107784
960	921600	884736000	995	990025	985074875
961	923521	887503681	996	992016	988047936
962	925444	890277128	997	994009	991026973
963	927369	893056347	998	996004	994011992
964	929296	895841344	999	998001	997002999
965	931225	898632125	1000	1000000	1000000000

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